Voltage – Current - Resistance - Ohms Law: Circuits (fall 2022)

You probably remember Ohm's Law as $V = IR$ or its algebraic identical twin: $I = V/R$. This seemingly simple formula packs in a lot relating just 3 variables. However, one must be very careful to properly apply it.

To wit: What is voltage V above? What is current I? How do we define them intuitively? Rigorously and quantitatively?

The questions below are meant to help build and guide your intuition and understanding, how to intuit and properly apply Ohm's Law. Once you lock in a few concepts, Circuits analysis becomes much easier!

Losing (?) your Marbles

1. Consider the 4 scenarios illustrated.
    a. Which way does the marble roll in each case? Intuitively why?
    b. Approximate the height of each end of the ramp (a and b) relative to 'ground' (0 inch mark). Now define that intuition in more rigorous physics terms, e.g. think about forces, energy, etc.
    c. Describe the analogy between this mechanical scenario and electrical circuits concepts? What is the physical quantity of interest at play in Circuits land? What are the 'rolling marbles' in an electrical circuit?

Pipe Dreams

1. Consider the 4 scenarios illustrated. The pressure at each end of the pipe is listed.
    a. Which ***direction*** does fluid flow in the pipe in each case? Intuitively, why?
    b. Again, convert your intuitive reasoning into more rigorous physical terms (if you haven't already!). That is, analogous physical quantity of fluid flow in an electrical circuit?
    c. In which case would fluid flow the fastest? Equivalently, in which case will the most water molecules per second flow? Assume all other factors are equal: same pipe diameter, length, material, etc.
    d. What is the analogous situation in Circuits land?

Dropping In

1. Consider the array of pipes shown at left, arranged in order of increasing diameter. Assume each pipe is subject to the same pressure difference, as illustrated.
   a. Which pipe will increase the highest volumetric flow rate? The lowest? Briefly justify.
   b. Construct an analogy in terms of electrical circuits.
2. Consider the 'Disk Drop' game at right (essentially a knock-off Plinko board from *The Price is Right*!)
   a. How does the speed of the yellow chips dropping here compare to a board with no pegs? With only ½ as many pegs?
   b. What is the analogy to Circuits, especially in light of Ohm's Law?

Ohhhhmmm's Law

1. Given all your thoughts and responses to the above, complete the expression for Ohm's Law in the box, very carefully filling in the numerator and denominator.
2. Consider the 4 scenarios illustrated. Here we see an actual resistor (you will see many in the near future in the lab!). Apply Ohm's Law compute the missing voltage or current.

Split Decision

1. Consider the parallel pipes configuration illustrated at left. Imagine there is an observer assigned to each pipe 1- 4 counting the volumetric flow rate (number of fluid molecules passing by per unit time). Assume each has an on/off valve. Let's compare 2 scenarios: i) Only valves 1-2 are open/fluid flowing (2-4 are closed) – VS - ii) all 4 valves open.
   a. In which scenario is the total volumetric flow rate higher?
   b. In which scenario is the relative proportion of fluid flow *per pipe* higher? Briefly explain.
   c. What is the relation between Q and Q_{1-4}? Write and equation and briefly explain.
   d. What does this imply about resistors in parallel? E.g. you might imagine 2 resistors in parallel vs 4 in parallel.

2. Consider the series pipe configuration illustrated at right.
   a. What is the relation between the volumetric flow rate in each pipe segment 1-3? E.g. write an expression relating Q1, Q2, Q3.
   b. Now let's compare 2 scenarios: i) Volumetric flow rate (fluid molecules per unit time) with all 3 pipes connected; ii) flow rate through pipes 1 and 2 still connected, but pipe 3 disconnected. In which case would the flow rate expected to be higher? Briefly justify.

   c. What does this imply about resistors in series?

3. Draw analogous electrical circuit configurations. The using your responses to 1c and 2a in this problem/section. What is the analogous relation between the various electrical currents I_x flowing in the system. This hints at Kirchoff's Current Law (KCL)!

Walk in the Woods: another kind of circuit

1. Let's look at a nice circuit hike. What is a circuit? It's just completing a loop, starting ad ending at the same point. Assume a hiker travels the loop a-b-c-d-a. (BTW, this is the Mt Pleasant Circuit, a beautiful ~ 5mi circuit hike about 40 min drive from Lexington. Highly recommended anytime, especially when autumn colors are on display circa mid-late October!).
   a. What is the change in gravitational potential energy moving from a to b? From b to c? From c back to a?
   b. What is the total change in gravitational potential energy from the start to stop point, marked a?
   c. What does this have to do with Circuits? This hints at Kirchoff's Voltage Law (KVL)!

The Punchline:

1. Ohm's Law: What is the 'V' in V = IR?
   a. It represents the ***difference*** in electrical potential between <u>**2**</u> points: $V_a - V_b$.
   b. Formally, voltage is the work per charge to move the charge between 2 points: $V = \frac{dW}{dq}$
2. What is the 'I' in Ohm's Law?
   a. It counts the ***charges*** per unit time flowing by a ***single*** point
   b. Formally, $I = \frac{dq}{dt}$
3. Kirchoff's Current Law:
   a. Charge conservation: current into a ***node*** = the current flowing out of a node
4. Kirchoff's Voltage Law:
   a. Energy conservation. Going all the way around a loop, energy gains = energy losses.
   b. Gains: power sources, batteries, anything adding or providing electrical energy to the system
   c. Losses: typically resistors (Joule heating); charges stored on capacitors, anything taking away or converting electrical energy into another form (e.g. LEDs convert electrical energy into photons)

BONUS PROBLEM – Audio volume control

If you've already sped ahead and find yourself feeling good about Ohm's Law, KVL, KCL, try solving this circuit. It is a fancy volume control for audio equipment. The goal is to write Vout/Vin as a function of α, which denotes the position of the potentiometer. To put this circuit in context: the higher Vout, the higher the volume. You should see the ratio Vout/Vin depends on the ratios of resistances in the circuits, hence α. To help visualize the result, plot your result for Vout/Vin using values of R1 = 1 kΩ; R3 = 22 kΩ; R2 = 100 kΩ potentiometer.