

Assignment # 3
 ENGN/PHYS 207—Fall 2022
 Making a Difference: The Wheatstone Bridge

Circuits You'll Build

1. Thermal Monitoring System

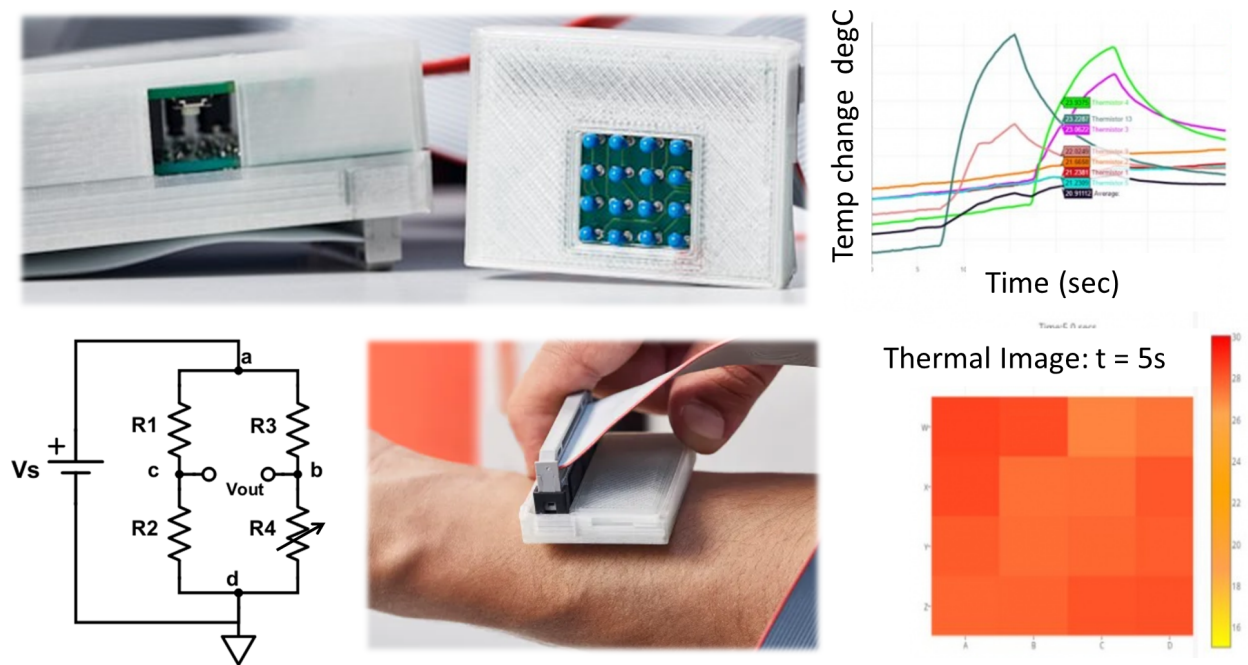


Figure 1: The *Skan* system for early detection of skin cancer. The design is based on an array of thermistors (array of blue dots in upper middle) configured in a Wheatstone bridge (bottom left). Temperature vs time data is converted into thermal images (right column). Image montage credits: see main text below.

Today we'll build a sensitive temperature measurement circuit using a thermistor configured in a Wheatstone Bridge. We're not the first and certainly won't be the last. The *Skan* device is one really interesting application. It won the James Dyson award in 2017 as a healthcare device developed specifically for lower resource settings. Here's a bit more about it via [alphr health](#), [the The James Dyson Award](#), and [Medium.com](#).

The *skan* device, *created by four engineering undergraduates* at McMaster University, Canada, works by measuring the temperature of a person's skin. Cancerous cells have a higher metabolic rate than normal cells, which means that if an ice pack is pressed on the skin, for example, the cancerous tissue will regain heat quicker than the rest of the skin. Using an array of **thermistors** – known as a transducer – the device is able to convert heat differences into electronic signals, and create a heat map of a person's skin after being cooled. These non-invasive temperature readings are then presented with a statement of findings about a presence, or lack of presence, of melanoma. Due to the difficulty of measuring an unknown resistance...a *Wheatstone bridge* topology was used.

Skills and Concepts You'll Learn

- Wheatstone Bridge principle and practical operation
- All about thermistors—aka temperature sensitive resistors
- More Arduino programming and interfacing

1 The Wheatstone Bridge

1.1 Introduction

Today we'll have a play with the classic **Wheatstone Bridge**¹ shown in the bottom left panel of Figure 1. The bridge is a clever 4 resistor configuration that **precisely measures small changes in resistance**. In essence it consists of *two* voltage divider circuits (can you spot them?). The bridge output subtracts the voltage dividers respective outputs at nodes b and c.²

While it was invented circa 1833 (people have been clever for a very long time...), it still finds widespread use today in many engineering applications: strain gauge vibration measurement in bridges and airplane wings; creating Hyperpod Thermal Imaging thermal images of futuristic transportation vehicles; controlling the temperature of your CPU and GPU as well as the ice cream freezer; and the thermistor, of course, plays a starring role in the venerable skin-contact digital thermometer.

Today, we'll build a thermal warning and or basic imaging system that implements a Wheatstone bridge configuration. In the near future, we'll build bridges—the sensors and the social structures—with the YMCA after school program!

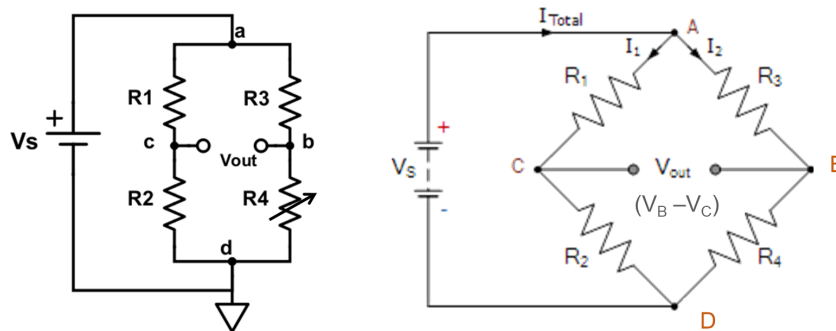


Figure 2: Basic Wheatstone Bridge Circuit drawn in 2 equivalent forms. The voltage source V_s provides excitation at nodes a and d. R_4 is a variable resistor. The output is measured as the voltage difference at node b relative to node c. The 'diamond' shape is very commonly shown out in the wild (image adapted from: <https://www.electronics-tutorials.ws/blog/wheatstone-bridge.html>)

The bridge consists of four resistors in a series-parallel configuration, “excited” by a constant voltage source V_s (e.g., power supply or battery). The output of the circuit is measured as the

¹Interesting historical side note: Wheatstone didn't actually invent this circuit. Credit for the first description of the circuit goes to S.H. Christie; but Wheatstone is the one who found widespread practical use for this circuit.

²At this point, you might wonder “Why use a WB instead of a simple and beloved voltage divider? Great question! A full discussion can be found on this excellent post at EE world; scroll down to “I still don't see the cancellation effect, can you explain?”

voltage difference across nodes b and c :

$$V_{out} = V_b - V_c = (V_b - V_d) - (V_c - V_d).$$

Note that node d is marked as **ground**, the point in the circuit designated at a fixed 0 V. The bridge is said to be **balanced** when $V_{out} = 0$. Starting in balance, changing the value of R_4 will make $V_{out} \neq 0$. In practice, R_4 might be a strain gauge, flexible resistor, thermistor—anything that changes resistance in response to some physical or environmental variable. A small value output voltage resulting from a small change in resistance is easily and accurately sensed. Why? Because even a small output voltage, say $V_{out} = 50$ mV, is a large percent difference away from 0 V. The same cannot be said of a voltage divider as the “balance” point would typically be $V_{out} = \frac{1}{2}V_{in}$. Thus, a 50 mV change away from 2.5 V is a very small percent difference, thus such a small change is much more difficult to accurately discern.

1.2 Theoretical Considerations

1. The bridge output is defined as the voltage difference between nodes b and c : $V_{out} = V_b - V_c$. On which respective nodes would you place the (+) and (-) probes of the volt meter?
2. There are 2 voltage dividers present in the circuit. Thus there are 2 complete loops for current to flow. Sketch them.
3. Show that:

$$V_{out} = \frac{R_1 R_4 - R_2 R_3}{(R_1 + R_2)(R_3 + R_4)} V_s \quad (1)$$

Hint: Work with the two voltage dividers separately, then combine them.

4. The bridge is said to be **balanced** when $V_{out} = 0$. From a design standpoint, the goal is to start with a balanced bridge: the system is in equilibrium about some known fixed point. For example, today our bridge output should read 0 V when the thermistor is at (near) room temperature. What is the relation for the resistances R_1, R_2, R_3, R_4 when the bridge is balanced?
5. Assume all resistors in the bridge are initially equal to the same value R . This is one special case of the bridge in balance ($V_{out} = 0$). Next, let R_4 increase its resistance by a relatively small amount: $R_4 = R + \Delta R$. (The other resistors are constant in value: $R_1 = R_2 = R_3 = R$.) For the case that $\frac{\Delta R}{R} \ll 1$, show that:

$$V_{out} \approx \left(\frac{V_s}{4} \right) \left(\frac{\Delta R}{R} \right) \quad (2)$$

“What’s the point of this math exercise?” you fairly ask. It shows the approximate linearity of the bridge output V_{out} with percent change in resistance away from baseline $\frac{\Delta R}{R}$. This makes calculations easy, though it isn’t that much more work to just use the full form in Eqn 2.

6. Now let’s do some quick plots and calculations to illustrate that the bridge reigns supreme compared to the venerable voltage divider for sensing small resistance changes. Assume the voltage divider consists of the power source, R_3 , and R_4 only, with the output measured

at node b relative to ground. Let's say the bridge is excited by a 5V power source and all resistor values are initially $R = 10\text{k}\Omega$ (shockingly similar to what you'll build in lab, fancy that) . Then let R_4 increase in value by 5%, or 500 Ω . That is $R_4 = 10500 \Omega$. Compute the expected output voltage for both the bridge and voltage divider. More importantly, compute the *percent change* in output voltage relative to the balanced condition. To make this calculation realistic and avoid infinity for the bridge, assume a realistic initial state of $V_{out} = 10 \text{ mV}$. Therefore, which circuit—bridge or divider—is more useful for accurately detecting small changes in resistance?

7. Lastly, use MATLAB to plot V_{out} vs. ΔR for both the bridge. Plot both the full form (Eqn 1) and linear approximation (Eqn 2). This plot will help guide your thinking and be used for comparison to empirical data you acquire in the lab!

2 Experiments/Circuit Design

2.1 Build a Bridge

“Is the section title really involve The WB?,” you ask. Good news: No Dawson’s Creek, Buffy the Vampire Slayer, or other low-quality re-runs here. Just the good ole Wheatstone Bridge.

1. Build the bridge (see Fig. 2). Use the classic 5 V and GND for V_s . Use 10 k Ω resistors for R_1 , R_2 , and R_3 . (Remember to carefully measure and record the actual resistance of each of these.) For R_4 , use a ≈ 20 k Ω pot. Balance your bridge by adjusting the value of R_4 . Carefully measure and record the resistance of the pot with your circuit balanced. This is the reference value against which you compute the change in resistance. For example, if R_4 is 9500 Ω when the bridge is balanced, then you turn the pot value to be 9400 ohms, this is means $\Delta R = -100 \Omega$.
2. Imagine R_4 to be an environmental monitor, perhaps a soil moisture gauge whose resistance changes with water content. Or perhaps R_4 represents a strain gauge element attached to an airplane wing to measure its vibrations during flight. If the wing flexes up- or downward, the resistance will change by an amount ΔR . Or maybe its a light sensitive resistor to measure subtle fluctuations in light intensity for an optics experiment.

We’ll simulate this for now by turning the dial on the ≈ 20 k Ω pot. Sweep the pot through a range of resistance from 5000 – 15000 Ω , in equal increments of $\approx 1000 \Omega$. This gives you about 10 data points total. Carefully measure, record and make a plot of V_{out} vs. ΔR . Remember: ΔR is the *change* in the resistance of R_4 relative to the balanced condition. (See above example in part a)

3. Overlay your data points on the voltage output versus change in resistance plots you previously generated in Part I: Theoretical Considerations illustrating the expected Wheatstone bridge behavior. Crucially, how do your empirical data compare with theory—be quantitative!

2.2 Hot and Cold: Thermal Monitor System

Now that you've seen a bridge in action, let's build an actual thermal monitor. The core idea is to use a thermistor integrated into the Wheatstone bridge. A thermistor is an circuit element with a temperature-dependent resistance. If only Isaac Newton had one back in the day! (See Figure 4). An example build from more recent times (courtesy of two illustrious alums, Taylor Witherell '20 and Emily Hassid '21) is shown in Figure 3. Notice the nice, simple wiring job making good use of a small space on the breadboard. The circuit schematic is very helpful in pointing out where connections to the Arduino should be made for measuring voltage. Last but not least, look at that beautiful temp vs. time data in the real-time plot!

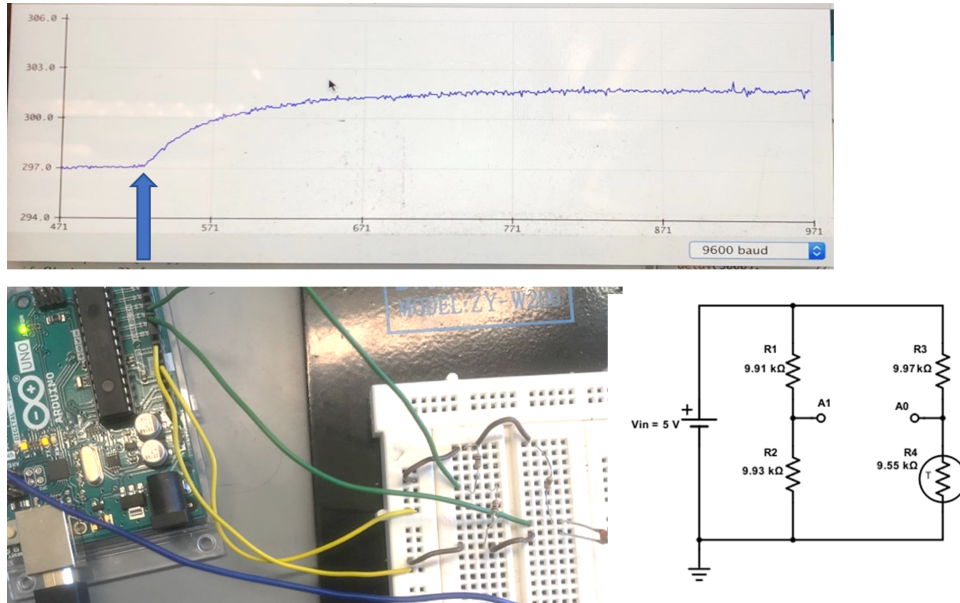


Figure 3: Example thermal monitor system. The arrow on the real-time plot indicates when the heat source (human finger) was applied to the thermistor, raising the temperature from about 297 to 301.5 K.

The **system design objectives** at hand are:

- Build a thermal monitor system based on a thermistor incorporated into a Wheatstone bridge.
- Use an Arduino to compute and show a real-time plot of temperature.
- Use Arduino to illuminate a LED when the temperature exceeds a pre-defined threshold of say 37C (required).
- **Optional** bonus fun: Make a more sophisticated LED display! Options you might consider:
 1. Vary the LED brightness proportional to the temperature. This requires 2 lines of code. See the short, sweet, and informative `Arduino analogWrite` tutorial [here](#) and `Fade` tutorial [here](#). Sample code provided!
 2. Control multiple LEDs at once. Maybe have a blue LED illuminate for colder temps; green for mid-range; and red for high temps. Here is a nice tutorial from the somewhat dubiously named [The Robotics Back-End](#).



Figure 4: Newton’s Lab Fire (top) and thermistor element (bottom). The legend goes that Newton’s dog knocked over a candle, setting fire to his experiment notebook. Another account says that he left a candle burning whilst he stepped out for a moment. A gust of wind blew it over, starting a fire in his lab. If only he had a thermal warning system! Image source: <https://www.alamy.com/stock-photo-isaac-newtons-lab-fire-18th-century-135096253.html>

The roadmap in Figure 5 below highlights the major steps involved going from voltage measurement to temperature output.

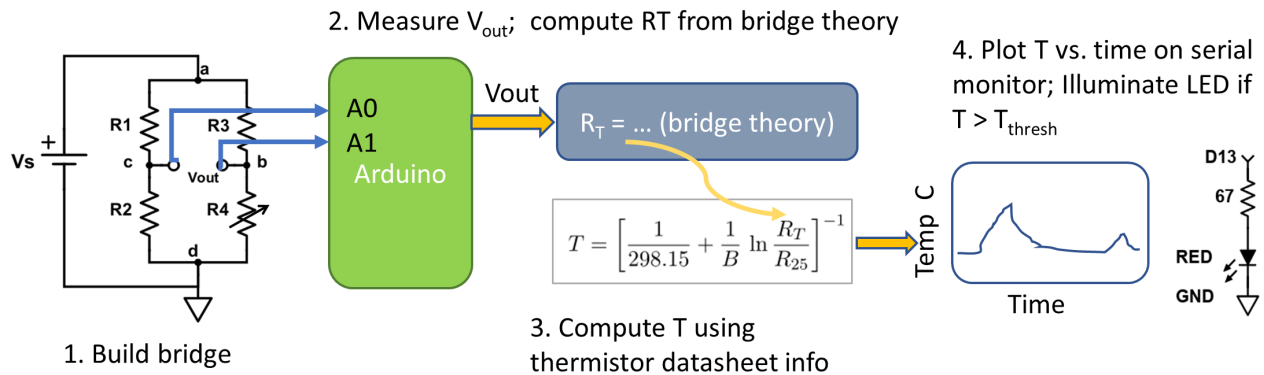


Figure 5: Thermal monitor system roadmap in 4 easy steps.

We’ll be using a 10k NTC thermistor purchased from Sparkfun (datasheet linked here). The “10k” indicates a nominal resistance at room temp (≈ 25 C or 298 K) is $R_{25} = 10$ k Ω . NTC stands for “negative thermal coefficient” meaning the resistance decreases with increasing temperature. From the datasheet³, we know that to first approximation the thermistor resistance R_T (units of Ω) as a function of temperature T (units of K) is given by:

$$R_T = R_{25} \exp \left[B \left(\frac{1}{T} - \frac{1}{298.15} \right) \right] \quad (3)$$

³See page 4, FORMULAE TO DETERMINE NOMINAL RESISTANCE VALUES

With a little bit of algebra, we can rearrange Eqn 3 to solve for the temperature (units of K) as a function of resistance, given in step 3 of Figure 5.

The B value (in units of K) is a material constant that can be found in the manufacturers data sheet (linked above or see course website). Look for the table row where $R_{25} = 10000 \Omega$. For the curious cat, a further explanation of thermistor resistance vs. temperature can be found on page 4 in a technical note supplied by the manufacturer, Vishay.

For **testing and validation** purposes, you can use your fingers for a gentle heat source. For more fire power (pun intended ha!), use a heat gun. Be careful and mindful while using it not to melt or destroy other components. An ice pack works well as a cold source. Be sure there isn't too much condensation on the surface, else you risk a nasty short circuit!

For the **high temperature indication light**, use a red LED in series with a current limiting resistor of about 100Ω . Carefully reiew the world-famous Blink tutorial for how to control an LED using and Arduino.

For **real-time plotting** of the temperature vs. time, consult one or both of these guides (depending on which version of the Arduino you installed). Read carefully, so you understand what text format the Serial Monitor can use to generate a plot.

1. Excellent tutorial from Adafruit demonstrating the Arduino Serial Plotter feature in Arduino IDE v1.x.
2. Crisp and easy to follow official Arduino documentation for IDE v2.0

2.3 More Arduino: Some tips on code for the thermal monitor system

Below you'll find some tips on programming and using the Arduino, specific to the experiment d'jour.

1. Use the 5V and GND power ports on the Arduino to power the bridge.
2. You've previously very successfully measured analog voltage on a single channel and displayed the result in the serial monitor before on a single port . Build on that!
3. Remember the bridge output is measured as the *difference* between the voltages at nodes b and c. Each Arduino analog input measures a single node in the circuit relative to ground. All measurements are made with respect to Arduino's ground (GND). So you'll need TWO separate analog reads to measure the bridge output V_{out} . For instance, connect node b on the WB to Arduino pin A0; and connect node c on the WB to Arduino pin A1.
4. Remember the `analogRead()` command reads in a 10-bit number (0-1023). Stupid computers, they only know 0's and 1's. We want an actual voltage. Furthermore, we know a reading of 1023 corresponds to 5V and a reading of 0 corresponds to 0 V. So there is a conversion factor of $5.0/1023$ going from a 10-bit analog read to actual voltage. So you might do something like this in your code:

```
int nodeB = analogRead(A0);
int nodeC = analogRead(A1);
float Vb = nodeB*(5.0/1023);
float Vc = nodeC*(5.0/1023);
float Vout = Vb - Vc;
```


See how easy it is to program the Arduino? :)

Note: lines of code such as the `float Vb` type tells Arduino “Please declare a 32-bit block of memory to store a variable we are calling `Vb`.” An `int` declares a 16-bit block of memory and assign it a value of what’s computed on the RHS of the equation.

5. You’ll still need to write a few more lines of code to compute the change in resistance ($\Delta R/R$), based on the WB output voltage (V_{out} being measured by the Arduino, and also compute the temperature. Note: the `log()` function in Arduino and C++ computes the natural log. The `log()` function is not native to the Arduino core function library, so you’ll need to include the math library. Toward the very tippy top of your code, you’ll need to do this:
`#include <math.h>`

This tells the compiler, please also use the math library so we can use functions like `log()`. Note the use of the hash tag (the ORIGINAL use, take that Twitter bird!). Also, note there is NO semicolon termination.

3 What to Turn In

3.1 Basic Wheatstone bridge Experiment with Potentiometer

The items below must all fit on 1 page maximum. Keep it clean, clear, correct, concise.

1. Beautiful plot illustrating Wheatstone bridge theory versus experiment.
2. Short paragraph commenting upon and *quantifying* theory vs. experiment. Did the bridge behave as expected? Across all values of ΔR tested?

3.2 Thermal Monitor System

This section may be a written or video submission (but not a mix). Videos are especially nice to show off live demos! Written submissions are limited to 2 pages max. Videos may be up to 3 minutes max.

1. Introduction highlighting the purpose of the circuit and intended applications.
2. Methods/Circuit Design highlighting the main principle of operation (“the forest”) and design rationale for actual circuit components selected (“the trees”).
3. Beautiful graphic(s)/illustration(s) showing proof-of-concept validation for your thermal warning system. Write a few words guiding the reader’s eye to what is shown.
4. Brief discussion section highlighting benefits and and limitations of your current thermal monitor device. How would you improve a future version of the system?
5. Arduino code in separate appendix, single-spaced (does not count toward page limit)!