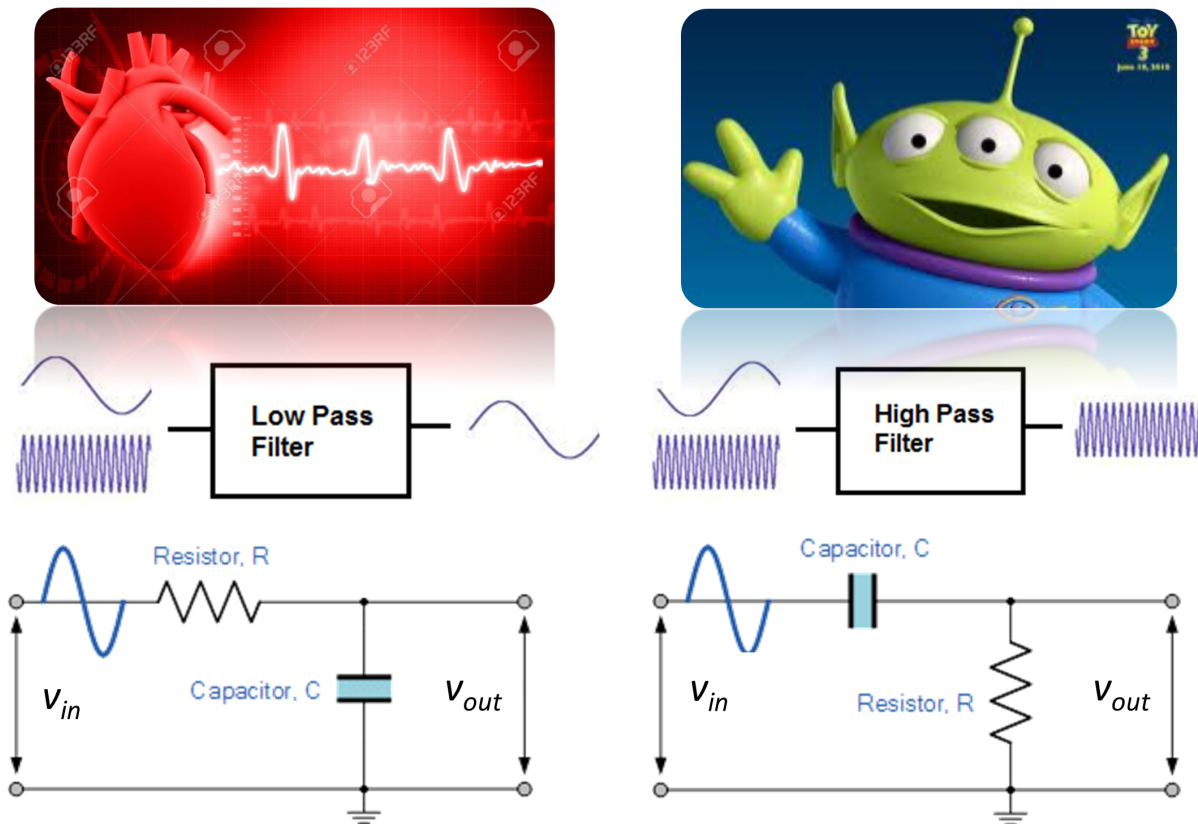# Circuits You'll Build/Design Problem Options

Figure 1: First-order Low-pass and High-pass filters. These are the simplest kind of RC filter. We'll use them in a biomedical and speech processing context. Image adapted from learningaboutelectronics.com and electronics-tutorials.ws at these four links (visit them, they are great resources!: [1], [2], [3], [4]

Today we'll build **RC filters** with application heartbeats and aliens talking. Strange, but true. If you are currently thinking, "OMG, it's just a voltage divider with a capacitor replacing one resistor", you'd be right! The **first-order** (or 1-stage) filters shown in Figure 1 are the most basic type of RC filter. You may cascade multiple of these to make a stronger filter, e.g. 2nd order. **You will be asked (assigned) to tackle only ONE of the design problems below**—either a low pass filter to isolate the ECG waveform OR a high pass filter to isolate aliens talking. Of course, you are most welcome to go for both, if you are feeling especially eager and energetic.

Skills and Concepts You'll Learn

1. How to use an oscilloscope and function generator—two classic pieces of test equipment in the circuits lab

2. Designing and building passive RC filters—they are classic and ubiquitous

3. Measuring the magnitude response aka decibel gain $G(f)$.

Filter Maths

Table 1 summarizes the frequency response of the simplest possible (yet highly effective) RC filters. The key parameter to any filter is the **cutoff frequency**. The cutoff (angular) frequency (rad/sec) is:

$$\omega_o = \frac{1}{RC} \quad (\text{rad/sec})$$

We humans think much better in cycles per second and its often much easier to use the more intuitive version in units of Hz:

$$f_o = \frac{\omega_o}{2\pi} = \frac{1}{2\pi RC} \quad (\text{Hz})$$

Also note that

$$\frac{\omega}{\omega_o} = \frac{f}{f_o}$$

as the factor of 2π cancels top and bottom.

Filter type	Transfer Function $H(\omega)$	Magnitude Response $\lvert H(\omega)\rvert$	Phase Angle
Low Pass	$\frac{1}{1+j\omega/\omega_o}$	$\frac{1}{\sqrt{1+(\omega/\omega_o)^2}}$	$-\tan^{-1}(\omega/\omega_o)$
High Pass	$\frac{j\omega/\omega_o}{1+j\omega/\omega_o}$	$\frac{\omega/\omega_o}{\sqrt{1+(\omega/\omega_o)^2}}$	$\pi/2 - \tan^{-1}(\omega/\omega_o)$

Table 1: Transfer function and magnitude response for 1st order low pass and high pass RC filters. For an N-order filter, *multiply* the magnitude response N times and *add* the phase angle N times.
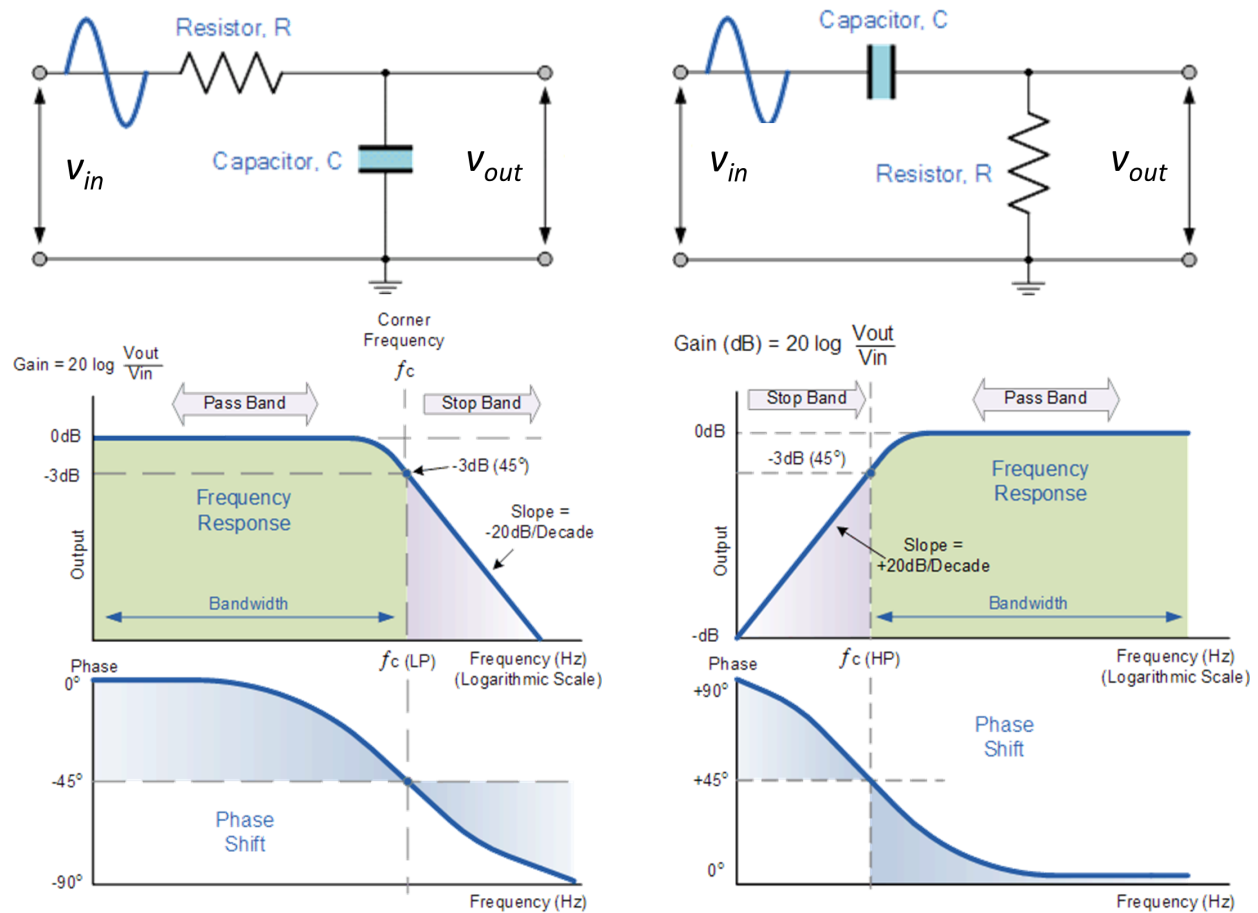
Figure 2: First-order Low-pass and High-pass filters frequency response for gain and phase. Image adapted from same refs as in Figure 1

.

1 Communicating with Aliens

(a) Alien the "claw has chosen",

(b) Buzz Lightyear ready for blastoff

Figure 3: Two Toy Story icons: the chosen alien and Buzz Lightyear.

Did you hear about the recently unearthed "secret" track from the silver screen smash hit, Toy Story? One of the Toy Story aliens has been "chosen by the claw" (c.f.: `https://youtu.be/zR3BLM_TAmg?t=140`). The cuddly lil' green guy is peacefully departing with some final words, an oratorical offering of solace to his fellow aliens—in the ever familiar fairly high-pitched Toy Story alien voice. But Buzz Lightyear, ever the proud show-off, will have none of it! He decides to save the alien from the claw. Immediately, he fires his jet pack, blasts upward and tries to snatch the alien from The Claw[1]. Amidst all of the commotion, another alien was recording for posterity the final words of the dearly departing friend. Houston (Lexington?), we have a problem! The low frequency, dull rumbling noise of Buzz's rockets are so loud that they nearly completely drown out the aliens parting words. Buzz Lightyear to the rescue? Hardly. Circuits students to the rescue? Now we're talking!

Your mission is to design and analyze the performance of an RC filter that selectively reduce the rocket's noise of the rockets and make clearly audible the alien's parting words.

A previous acoustic analysis of Buzz's jet pack (rumor has it SpaceX/NASA did the analysis) indicated most of the sound content occurs at frequencies ≤ 300 Hz. Similarly, the aliens speech content is suspected to be concentrated at frequencies ≥ 500 Hz. Because of the frequency needle to thread, so to speak, is very small (recall factors of $10\times$ are what's important in frequency space), you may want to consider a 2nd order filter for this application.

One helpful tip based on subtle but important knowledge of our equipment: Use an $R \geq 5.6$ kΩ resistor in your design[2]. The function generators in lab typically have an output impedance of 50 Ω, so they can also safely drive the filter without signal loss.

---

[1]OK, Toy Story aficionados, this is heresy, I know. It's not really how the movie plays out. The original plot line has been modified to fit conveniently into the Circuits pedagogy.

[2]This is a nice mid-range value resistor. Plus, the picoscope has an output impedance of 600 Ω, so we can only "drive" loads that are about $10\times$ larger, e.g. 6 kΩ, without voltage division becoming too severe.

1. First and foremost, do a test run with a few variations of your filter. Verify via proof of demonstration that your circuit performs the job it is intended to do! You may need to consider a 2nd order filter, as necessary.

2. Sweep out the magnitude response vs. frequency for your filter. The frequency range you test should be at bare minimum one ***decade*** (factor of 10) above and below the cutoff frequency (40 to 4000 Hz, in this example), better if you can push to 2 decades above and below. Do be aware that 100 kHz is the maximum frequency that can be generated by the Picoscope 2204A AWG (arbitrary waveform generator). The function generators in the lab can go much higher into the MHz range.

3. Make a plot of the experimental ***decibel gain*** $G(f)$ vs $\log_{10} f$.

4. Overlay the theory curve so that you can instantly compare and contrast with experiment. Highly recommend matlab here, you even have sample code at your disposal. Show the instructor of TA your plot when you are done (BEFORE you walk out of lab).

5. When all looks good and sensible—proof of concept your filter is working properly has been established—we'll connect you into the audio so you can the net result for yourself. The audio file you need is on the course website titled "Aliens Talking with Rockets" linked here.

   To reiterate: in order to complete this lab, provide a live demonstration of the your circuit. After all, we are on the edge of our seats wanting to know what the secret message was!

2 Listen to Your Heart: Electrocardiogram (ECG)

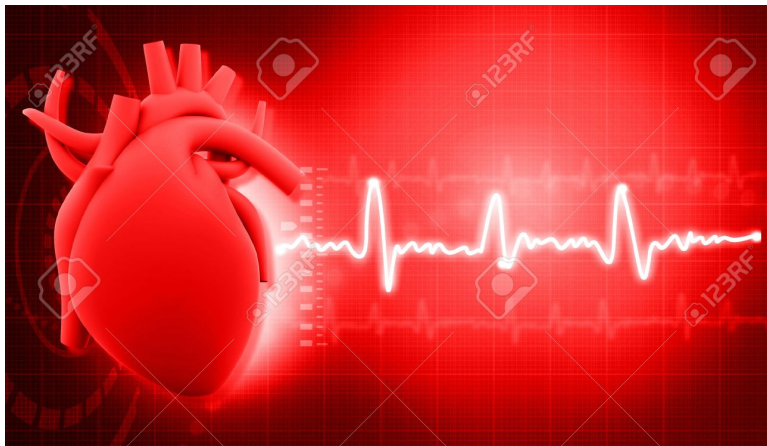

Figure 4: Heart beat muscular contractions produces an electrical signal. Image credit: 123rf.com

2.1 Intro to the ECG

The electrocardiogram (ECG) is a critical diagnostic tool to assess heart health in medical clinics worldwide. In essence, it is an electrical measurement of the heart's muscular activity pumping blood throughout the body.

One basic measure derived from the ECG is heart rate. The cleaner the ECG signal, the easier it is for a computer-automated algorithm to find the peaks of the QRS wave, and thus the more accurate the determination of the heart rate. A more sophisticated analysis of the ECG can reveal abnormal cardiac activity, such as atrial fibrillation (AF). Abnormal heart activity shows up in the ECG as abnormally timed and/or abnormally shaped waveforms. Again, it is absolutely critical to get a high quality ECG for the cardiologist to properly analyze!

Before plunging ahead, let's see how the heart beat generates an electrical signal on the skin surface. Watch the last 2 min of this very well done video that explains how electrical signals arise from muscle contractions. It also explains the origin of some interference signals that should be filtered out in order to recover the true underlying heartbeat signal alone. Good filtering leads to good quality signals. Good quality signals lead to accurate diagnoses–excellent!

All righty, let's see what we are working with today. The "clean" (non-noisy) signal, which we will call $v_{ecg}(t)$ was measured from an actual patient's human heart The ECG signal $v_{ecg}(t)$ was retrieved from the MIT ECG stress test database here. This signal is illustrated in 5, bottom panel.

High frequency noise, $n(t)$, was added synthetically inside a computer (Matlab!). The added noise added mimics real life hospital conditions with power line interference and high frequency medical instruments contaminating the clean ECG signal. Thus the total record signal is:

$$v_{noisy}(t) = v_{ecg}(t) + n(t)$$

The noise was generated using the following sum of cosines:

$$n(t) = a_1 \cos(2\pi f_1 t) + a_2 \cos(2\pi f_2 t) + a_3 \cos(2\pi f_3 t)$$

where the amplitudes are: $a_n = [50, 100, 25]$ mV, and the frequencies are $f_n = [180, 300, 4000]$ Hz.
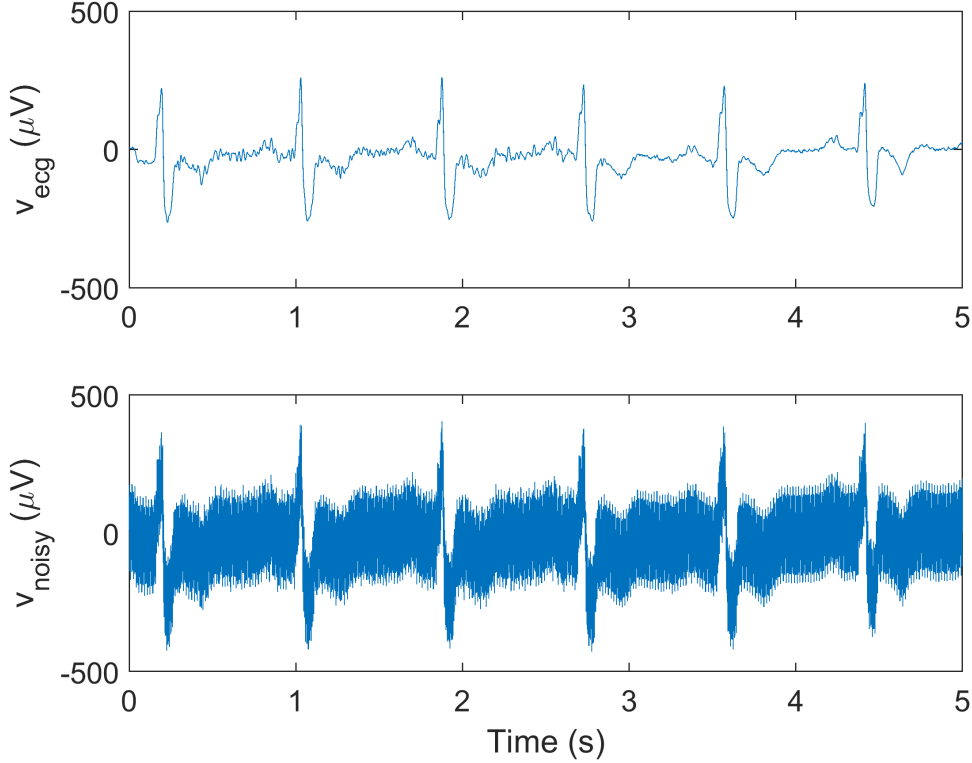

Figure 5: Top: Clean ECG signal. Bottom: Noisy ECG signal.

The QRS complex can be variable among patients. They typically have a duration of about 50 - 150 ms. You can think of this as the period of the underlying oscillation. Thus, we can also say that the underlying frequency range of interest spans $f = 1/150$ ms to $1/50$ ms. Equivalently, we can say the QRS complex typically is in the frequency range of $f = 6.7$ to 20 Hz.

To recap, the highest frequency we care to keep is about 20 Hz and the lowest frequency noise component is 180 Hz.

2.2   Problem Statement: Low Pass Filter Design

Dear fearless Circuiteers, doctors and patients need your help cleaning up the noisy ECG signal. Specifically, they need your new found expertise to build a low pass filter that sufficiently attenuates the noise and interference sources (frequencies of 180 Hz and higher), whilst preserving the underlying ECG signal QRS complex (frequencies of approximately 20 Hz and below).

Your task is to design, build, characterize, and perform proof of concept experiments for a LPF that does just this: ditch the high frequency noise, keep the underlying ECG signal. That is, don't throw the proverbial baby out with the bath water!
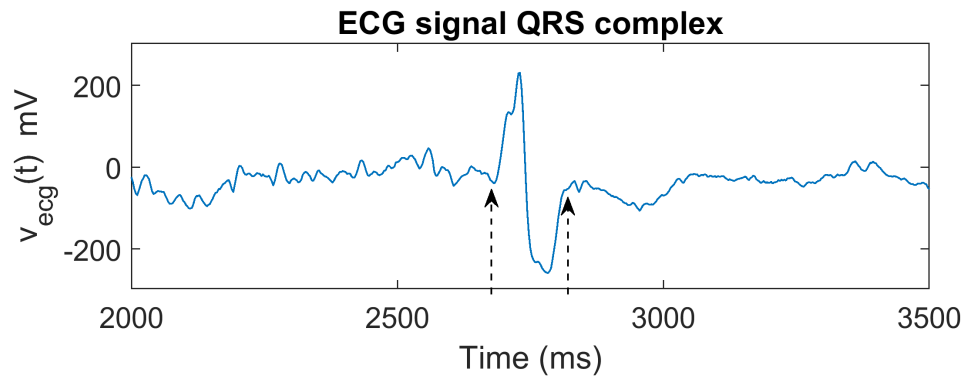
7

Figure 6: QRS complex timing of ECG signal in today's data set. The duration of the complex (period of underlying oscillation) indicated by the dotted arrows is $\Delta t \approx 130$ ms.

One helpful tip based on subtle but important knowledge of our equipment: Use an $R \geq 5.6$ kΩ resistor in your design[3]. The function generators in lab typically have an output impedance of 50 Ω, so they can also safely drive the filter without signal loss.

Some steps you should take to complete this design problem include:

1. Select a suitable cutoff frequency. Discuss/verify this with the instructor of TA prior to actually building.

2. Build and test your filter's performance, adjusting as you see fit. Experiment with 1 and 2 stage filters as necessary. Remember, the goal is to recover the underlying ECG signal per Figure 5.

3. Pipe in the ECG signal from your computer or mobile device. The ECG signal is formatted as an .wav audio file (easy to play back this way) from the course website called "noisy ecg" linked on Box here. (Note: this link was updated 1.23 pm 27 Oct. This .wav file is sampled at 44100 Hz. The previous .wav file available at this link was sampled at only 360 Hz, which was causing issues upon playback, basically time was stretched by a factor of $44100/360 = 122.5$ for some playback devices.)

   The clean ecg data file is also provided on the course website for reference. Note: If you try listening to these file, you won't hear a heart beat; rather you'll hear a hiss that corresponds to all the high frequency noise components.

4. View the noisy signal (input) and the clean(er) signal (filter output) simultaneously using your scope. Does it look better/cleaner? If so, great. If not, adjust to taste! When you are happy with the result, verify with the instructor or TA before moving on to the next part.

5. Time to fully characterize the magnitude response vs. frequency for your final filter design! That is, gather sufficiently data for plotting $G(f)$ vs. $\log_{10} f$ that you can compare against theory. Remember to work across decades (powers of 10) in frequency land and be careful to get enough data where things are changing around the cutoff frequency!

---

[3]This is a nice mid-range value resistor. Plus, the picoscope has an output impedance of 600 Ω, so we can only "drive" loads that are about 10× larger, e.g. 6 kΩ, without voltage division becoming too severe.

6. Plot your data overlaid on the theoretical curve.Verify all looks sensible—check with the instructor or TA.

3 What to Turn In

Each individual student needs to turn in their own report.[4]

 2 pages max allowed! You should basically be done with this report as you walk out of lab. There is not much writing, the figures should tell the majority of the story.

1. State what the practical purpose of your filter is. (1 sentence should suffice)

2. Concise rationale explaining your filter design, for example: "I designed a (fill in the blank) filter with a cutoff frequency of X Hz because (supply reason here)." Show any relevant calculations, e.g. cutoff frequency given R and C value

3. Clearly labeled circuit schematics, all component values clearly labeled must be done using a web app/software—not hand drawn.

4. Beautiful plot of decibel gain experiment vs. theory. **_MUST BE DONE IN MATLAB._**

5. Proof of concept that your filter worked sufficiently well. Record, post, and link to a video as you deem appropriate (Aliens audio especially) or make a convincing before and after filtering figure (ECG).

6. Concise discussion of pros/cons of the design, for example: "My filter design worked well in that (intelligent comment goes here). One limitation or possible improvement for the design was (some aspect that was suboptimal goes here)."

---

[4]Rationale: Let's make sure everyone is building all the core skills: drawing circuit diagrams; using matlab to make pretty plots of theory vs experiment; scientific writing.