Wave Equation in 1-D: Separation of Variables
ENGN/PHYS 225—Winter 2021 (April 5, 2021)
J. Erickson

Foreword

We've covered a LOT of ground in math methods. Here, we reviewthe large building blocks, the big conceptual pieces you should take from this course that will serve you well throughout your time in the W&L Phys-Engn dept and well beyond.

Vector calculus

Know the physical interpretation of the following. That means you should be able to look at a scalar field contour plot or a vector field "quiver" plot and answer questions like "which way would a little pinwheel rotate here?". Know whether scalars or vectors or input and output. It isn't crucial to remember any formulae (though they aren't too difficult), but you know how to compute when needed

1. Gradient
2. Curl
3. Divergence
4. Laplacian

Complex numbers

Euler's ID is the key here. It is the most mathematically convenient form of all time in real-world applications. Because everything oscillates and/or oscillates with damping.

1. Euler's ID. Know it inside and out. Know the relation between trig (drawing triangles) and plotting points in the complex plane.
2. Know how to graphically interpret $z = re^{i\phi}$. Which of these sets the magnitude? Which terms controls the phase angle, thus may be interpreted as pure rotation of a point in the complex plane?
3. Know how to convert between forms of $z = a + ib = re^{i\phi}$
4. Manipulate expressions adding, subtracting, multiplying, dividing complex numbers (e.g., $z_1 + z_2$ or $z_1 z_2$).

1st and 2nd order ODEs

1. Guess and check solution method e^{rt}
2. Apply ICs
3. Be able to sketch solutions

4. Don't necessarily memorize formulae, but be familiar with undamped and damped oscillations.

5. Intuitively sketch solutions

Linear Algebra

1. Perform basic matrix operations, namely addition and multiplication

2. Cast systems of equations in matrix $Ax = b$

3. Recognize the ubiquitous eigenvalue/vector problem $Ax = \lambda x$ or equivalently $(A - \lambda I)x = 0$. Be familiar with how to solve these—what must be true of the determinant?

Fourier Series

1. General idea of Fourier Series and transforms of superpositing sine waves to build more 'complicated' functions

2. Know the two sides of the time-frequency coin. Given a simple Fourier spectrum, sketch the time domain signal and vice-versa.

3. Know what orthogonality means and why it is important

4. Rest assured, no one in their right mind would ask you to memorize formulae for Fourier coefficients!