Math Methods (Engn/Phys 225) Matlab workshop (02 Feb 2021)
Plotting surfaces, contours, and vector fields

1. **Launch Matlab** (available through myvi.wlu.edu)

2. **Download example code** .m files in this box folder (linked on course website; scroll down to "supplemental section"). There are 3 .m files. These are matlab *scripts* that open in the matlab editor. Save them to your own drive; you will modify and add to them. Also, keep this in an obvious spot so you can refer back to them in the future.

- **`ExampleCode Surface and Contror plots.m`**: plot surface and countour lines, also demonstrates fancy LaTeX formatted title text
- **`rotationalField quiverExample.m`**: plots vector fields in 2D and 3D.
- **`div color plot w quiver.m`*:*** plots vector field overlaid on a color-coded "heat map"

PRO TIP: Make a new .m file script for every problem you work on. Of course, you can and should copy and paste snippets you have used or seen previously.

3. **Surfing surfaces: Scalar function surface and contour plot**. In your problem set this week, you are asked to consider a fearless surfer bombing down a wave. The wave height as a function of position is given by:

$$h(x, y) = e^{x/a} + bxy^2 \text{ , where } a = 5,\ b = ½.$$

- Make a surface plot to render a 3-D visualization of $h(x, y)$. Key matlab function: `surf`. *Surfing surfaces, whoa gnarly!* It is recommended that you restrict the domain of the wave to something reasonable, for example $|x|<5$ and $|y|< 10$.
- Make a contour plot to visualize $h(x, y)$. Key matlab function: `contour`
- Overlay a vector field plot to visualize negative of the gradient $-\nabla h$. Use this result to trace the approximate the path Fearless Fred would follow if his quest was to follow the gradient on down to bottom. Key matlab function: `quiver`

5. **Sharpest tool in the math shed: The Laplacian**. In class, we discussed how the Laplacian can be used to sharpen images (photographs or electrical "images"). Let's say you have an image with intensity given by a 2-dimensional Gaussian function:

$$I(x,y) = a\, e^{-\left[\left(\frac{x}{b}\right)^2 + \left(\frac{y}{c}\right)^2\right]}, \text{ where } a = 4,\ b = 2,\ c = 3$$

- Make a surface or contour plot of $I(x,y)$. This should help you visualize the size scale of the Gaussian "bell shape".
- Develop an expression for the Laplacian $\nabla^2 I$
- Make a surface or contour plot of the $\nabla^2 I$ in a separate axis next to the one you made illustrating $I(x,y)$.
  Key matlab functions: `surf`, figure, subfigure

6. **Vector field overlaid with color-coded background image:** Swirly Twirly Gumdrops. In class, we had a play with Buddy the Elf's favorite vector field:
$\vec{v} = \cos(\pi(2x - y))\,\hat{\imath}\, - \sin(\pi(2x + y))\,\hat{\jmath}$.

- In the next step, you'll compute the curl of the swirly-twirly gum drops vector field. Before cranking the math wheel, make a quick intuitive argument why it is "obvious" that the vorticity $\vec{\omega}$ is (anti-)parallel to the z-axis. What does this mean physically?
- Write an expression for the curl $\nabla \times \vec{v} = \vec{\omega}$. This is often termed the ***vorticity***.
- In the next step, we'll overlay a plot of the swirly-twirly vector field $\vec{v}$ with a color-coded background showing the k-hat component of the vorticity ω_z. Key matlab functions: imagesc, quiver, meshgrid

7. **Field day: electric dipole field lines;** Problem 4.107 from the Felder & Felder textbook is a classic one from E&M. It deals with electric dipoles which pop up all over the place in physics class and in the real world. Solve problem 4.107 and use matlab to plot your results to visualize the equipotential lines of $V(x,y)$ and the plot the electric field $\boldsymbol{E}(x,y)$. Hint: you already solved this in class last week, remember $\vec{E} = -\nabla V$, and you know what the classic dipole field should look like