Sample Final Exam Questions
ENGN/PHYS 225—Winter 2021 (April 5, 2021)
J. Erickson

Foreword

Below are some (≈ 5) sample final exam questions. There will be 10 total on the actual final exam. These are intended to help you review major concepts and gauge what and how to review.

Sample Problems

1. Sketch the 2D vector field (imagine it to be a velocity field of fluid flow:

$$\vec{v}(x,y) = -y^2\hat{\mathbf{\imath}}$$

2. Figure 1 shows a 2 panel figure. Identify (mark on the plot) 1 region where the divergence negative and one spot where the curl is non-zero (For the curious STEM cat: This data shows how electrical waves travel in the stomach. Image credit: T. Wang et al; *Physiological Reports*, 2020

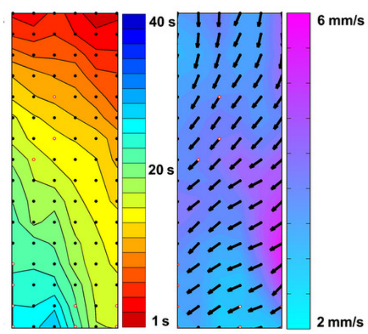

Figure 1: Image from: T. Wang et al, *Physiological Reports* (Dec 2020): "The influence of interstitial cells of Cajal loss and aging on slow wave conduction velocity in the human stomach"

3. Draw the point $z = 5e^{i\frac{\pi}{4}}(2+3i)$ in the complex plane. Also, write z in complex exponential form.

4. Sketch $p(t)$ vs. t, representing the change in pressure vs time, where $p(t) = Im[-3e^{j4\pi t}]$.

5. Given the following systme of equations:

$$\begin{aligned} 5x_1 - 4x_2 &= 10 \\ x_1 + x_2 &= 0 \end{aligned}$$

   (a) Write this system of equations in $A\vec{x} = \vec{b}$ form, where $\vec{x} = [x_1, x_2]^T$.
   (b) Can we find a unique solution for $\vec{x} = A^{-1}\vec{b}$. Briefly justify why or why not.

6. Wifi and Bluetooth transmit and receive information via radiowaves oscillating at a carrier frequency of $f = 2.4$ GHz (or $\omega = 2\pi f$ rad/s) As you likely know from experience, these wireless data transmission protocols aren't particularly good at penetrating through walls. Why? Two words: skin effect! This is a casual term that describes how electromagnetic field energy is absorbed and dissipated by the medium through which the waves travel.

   The skin effect fundamentally derives from Maxwell's equations writing:

$$\frac{d^2 B}{dy^2} = i\omega\mu\sigma B$$

   Here, $i = \sqrt{-1}$; μ and σ are positive, real valued constants (the permittivity and resistivity of the medium...metal, concrete, water, air, whatever); and B denotes the the amplitude of the magnetic field varying as a function of distance y.

   Phew, that was a long intro. The main point of this problem is this: you can solve it by just guessing everyone's all-time favorite guess!

$$B(y) = ce^{rt}$$

   Solve for the characteristic roots r; show they are complex numbers; define the skin depth δ in terms ω, μ, and σ. To reiterate, δ determines how fast the magnetic field amplitude exponentially decays with propagation distance y.

   Fun fact: plugging in values for sea water and an oscillation frequency of $\omega = 2\pi(2.4)$ GHz, the skin depth is only $\delta = 0.16$ m. That means you can expect to transmit Wifi signals, if you are lucky, over a distance of about 1 m only!

7. Figure 2 shows the frequency domain representation of a signal. Sketch the corresponding signal in the time domain. Offer a 1-2 sentence explanation of why you drew what you drew.

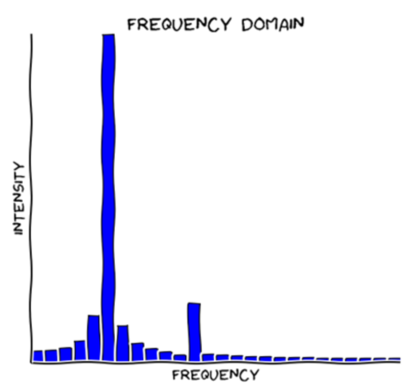

Figure 2: FFT. Assume the peaks occur at 0.5 and 1.0 Hz. Sketch the corresponding time domain signal. Image credit: `https://learn.adafruit.com/fft-fun-with-fourier-transforms/background`