ENGN/PHYS 225—Winter 2021
Integrative/Challenge Problem 1 Due Date: Monday, 22 Feb, in class

Introduction

Thus far we've explored the major vector field operations: gradient, divergence, curl, and the Laplacian. We've gotten some good physical intuition and learned how to crank the math wheel a bit. Now it is time to take a deeper dive and apply these to a real world problems!
Teams of (up to) 3 students will work together to develop a math solution and class presentation for one of the problems described below. Feel free to pick the one that piques your interest. There should be no boring math! If none of them tickle your fancy, you may propose a subject of problem for consideration; the instructor will work with you to formulate a problem with appropriate scope. Once you select a project and team, the instructor will provide further resources to help you approach the problem. Regardless of which problem you choose, expect this to be a challenging, rewarding, and fun exploration through vector calc!

Problems on Offer—Choose One!

1. **Field Day: E&M**

   There are quite a few fundamental models for electromagnetic fields: the point charge, the dipole, quadropole, infinite sheet of charge, corner of a cube, straight wire or loop with charge flowing. Pick three (or more) of these that interest you. Starting with the solution for the electric or magnetic potential, derive an expression for the the corresponding field $\vec{E}$ or $\vec{B}$. Visualize these fields by plotting contours and field lines. Compute the divergence and curl of each and visualize them as well. Interpret your results in context.

2. **Gut Check: Better health care through math** Recall from class we discussed how biological electrical sources inside the body—heart, brain, colon—produce a blurred "electrical image" when measuring the corresponding electric potential (aka voltage) on the skin surface. We further discussed how using the Laplacian can help sharpen the image to more definitively and precisely locate the underlying sources. In other words, a little bit of math can lead to better healthcare—hooray! In this challenge problem we'll explore the promise and the peril with this method, as well as fundamental limitations.

   Specifically, this problem will explore:

   (a) How close or far away the dipoles can be and still resolve them using the Laplacian image sharpening technique

   (b) Is there any benefit to using higher-order derivatives, e.g. $\partial^4 V/\partial x^4$

   (c) Noise amplification: what happens when we add noise to the system, does the Laplacian still work?

   If you are interested in biology/medicine and/or image processing, you might consider tackling this fun and exciting challenge.

3. **Need a Lift?**

   In this problem, we'll use vector calculus and borrow a theorem or two from fluid mechanics (Kutta-Jukowski theorem) to investigate how bats, birds, and insects are able to generate lift and thrust. The same phenomenon also explains how aquatic animals propel themselves ("flying through water"). Along the way, extend our vector calc toolbox with Stoke's and Green's theorems. The culmination will be to apply these math concepts to analyze data sets previously published by other labs. If you are like fluids and/or are fascinated by things that fly, then this problem is for you!

4. **Curveball! Fluid Mechanics of bending soccer and baseballs**

   This problem explores explores how and why rotating spheres tend to curve. Bend it like Beckham? You bet! In math terms, this problem takes a dive into the vector calculus involved with spinning balls. Along the way, you'll learn about the Magnus force, and the concept of potential flow. Ultimately, all of the solutions to describe fluid flow in these context of astonishing athletics are built around gradient, divergence, and curl. If you like mixing math and sports, give this one a go!

5. **Tornado Alley**

   Use National Weather Service/NOAA data sets to describe weather events. You might look at the pressure gradients driving weather patterns, or the curl or divergence of larger scale atmopsheric physics. Calling all weather junkies!