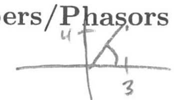
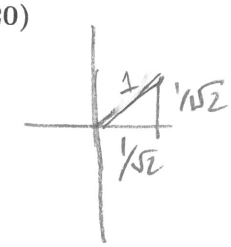
ENGN/PHYS 207

Complex Numbers/Phasors Worksheet (September 8, 2020)



1. Let's say we have two complex numbers: $z_1 = 3 + 4j$ and $z_2 = \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}$.



$|z_1| = 5$ $\angle z_1 = 53°$ $= 1 e^{j\pi/4}$

   (a) Sketch z_1 and z_2 in the complex plane.

   (b) Write z_1 and z_2 in the complex exponential form.

   (c) Compute and sketch: $z_3 = z_1 + z_2$.

   (d) Compute and sketch: $z_4 = z_1 z_2$. Hint: it may be very helpful to turn each of these into complex exponential form first.

   (e) Compute and sketch: $z_5 = \frac{z_1}{z_2}$.

2. Many times in circuits we are interested in ratios e.g. $R1/(R1 + R2)$. Similarly, we'll be interested in impedance ratios in the near future $Z_1/(Z_1 + Z_2)$. Let's get our feet wet with some basic expressions to see how complex exponentials save the day.

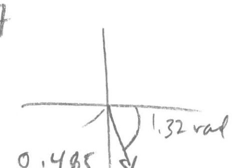
   For each of the following compute and sketch in the complex plane $\tilde{V}_{out}$ given by:

   (a) low pass filter output example:

$$\tilde{V}_{out} = 2\left(\frac{1}{1+4j}\right) = 2\left(\frac{1e^{j0}}{\sqrt{17}e^{j1.32}}\right) = \frac{2}{\sqrt{17}}e^{-j1.32}$$

   (b) high pass filter output example:

$$\tilde{V}_{out} = 2\left(\frac{4j}{1+4j}\right) = \frac{2\cdot 4}{\sqrt{17}}e^{j(\pi/2 - 1.32)}$$



   (c) band pass filter output example:

$$\tilde{V}_{out} = 2\left(\frac{4j}{1+4j}\right)\left(\frac{1}{1+0.5j}\right) = 2\left(\frac{4e^{j\pi/2}}{\sqrt{17}e^{j1.32}}\right)\left(\frac{1e^{j0}}{1.11e^{j0.46}}\right)$$

   (d) Lastly, assume that we are working with a wave that oscillates with a frequency $f = 5$ Hz. Sketch the time domain signal $v_{out}(t)$ for each case above.

$$= \frac{2(4)(1)}{(1.1)(\sqrt{17})} e^{j(\pi/2 + 0 - 1.32 - 0.46)}$$

$$= 1.76\, e^{j(-0.21)}$$

rad $\approx -12°$