Solution to in-class power problems
ENGN/PHYS 207 fall 2020

Solutions

1. **Back to the Future**

   (a) Current is charge flow per time:

   $$I = \Delta q/\Delta t = 5\text{C}/300\text{ms} = 16.67 \text{ A}.$$

   Note that I've written these as deltas instead of the full derivative, since we are basically interested in the initial and final times of the lightning strike, not all of the fine detail about time dependence during the strike.

   (b) The charge per electron is $e = 1.6 \times 10^{-19}$ C. Thefore, the number of electrons transferred is:

   $$N_{electrons} = \Delta q/e = 3.13 \times 10^{19}.$$

   McDonald's claims to serve 27 million people were day—12 orders of magnitude less than the number of electrons in a bolt. Pass the ketchup.

   (c) Work, voltage and charge are related by $V = \Delta W/\Delta q$. Since work = energy, we can rearrange to get:

   $$dW = V dq = (10\text{MV})(5\text{C}) = 50 \text{ MJ}.$$

   (d) Power, voltage, and current are related by:

   $$P = IV$$

   . We are given V, and we computed I in part a), so plug in to get:

   $$P = (16.67\text{A})(10\text{MV}) = 166.7 \text{ MW}.$$

   (e) Marty needs 1.21 GW to get "back to the future": `http://www.youtube.com/watch?v=f-77xulkB_U` Based on our answer above, poor Marty is gonna be stuck watching himself be erased from existence.

2. **A Bright Idea**

   (a) The power rating at 120 V means: "Take a 120 V battery, plug a single lightbulb into it, and here's how much power is dissipated." For instance, if you plug a 40W lightbulb into a 120V source, it will dissipate 40 W; but if there is a different voltage across it, it will dissipate something other than 40 W. Anyway, combine $P = IV$ with Ohm's law (applicable in this case, clearly) to get $P = V^2/R$. Rearrange to get $R = V^2/P$, where V is the voltage across the light bulb—in this case 120 V—and P is the power rating (at 120 V). Then plug in for each light bulb:

   $$R_1 = (120V)^2/40W = 360 \ \Omega.$$

   $$R_2 = (120V)^2/60W = 240 \ \Omega.$$

   $$R_3 = (120V)^2/75W = 192 \ \Omega.$$

(b) This is clearly a series circuit with total series equivalent resistance $R_s = R1 + R2 + R3 = 792\Omega$. Now the circuit is simplified to V_s (the battery) and R_s. Apply Ohm's law:

$$i_s = V_s/R_s = 100\text{V}/792\ \Omega = 126\ mA.$$

(c) In a series circuit, the current flowing through each bulb is the same, and equal to i_{total}, computed above. To compute the power dissipated across a resistor for which the current flow through it is known, use $P = I^2R$. Plugging in for each bulb, using the R's computed above, we get:

$$P_1 = i_s^2 R_1 = 5.7\ W.$$

$$P_2 = i_s^2 R_2 = 3.8\ W.$$

$$P_3 = i_s^2 R_1 = 3.0\ W.$$

Thus, the total power consumption is 5.7 + 3.8 + 3.0 = 12.5 W.

(d) Intensity is related to the amount of power dissipated. In this case, P_1 dissipates the most power, followed by P_2, then P_3. This seems like a counter intuitive result: The bulb with the lowest power rating is the brightest. That's not what you are used to. Why? Because in a normal situation in your house, everything is in parallel—everythign you plug in "sees" the full 120V provided by the wall outlet. For instance, if you have 3 lamps turned on, with different rated light bulbs, the highest power rating is the brightest.

(e) No, order doesn't matter. They are all in series no matter what. Thus, the total amount of current will be the same, hence the power ratings will stay the same, and so will the voltage drops across the bulbs.

(f) In a parallel configuration, each bulb has 100 V across it. Thus, we can compute the power using $P = V^2/R$.

$$P_1 = V_{R1}^2/R1 = (100\ V)^2/\Omega = 27.8\ W.$$

$$P_2 = V_{R2}^2/R2 = (100\ V)^2/\Omega = 41.7\ W.$$

$$P_2 = V_{R3}^2/R3 = (100\ V)^2/\Omega = 52.1\ W.$$

Summing up power dissipated in the parallel circuit we get $P_{parallel} = 121.5$ W. Since the series configuration consumes less power, the battery won't run out as fast, and it will remain lit longer (almost 10 times as long!).