Assignment 4: Analog-Digital Conversion
ENGN/PHYS 208—Winter 2019

Background

The physical world 'speaks' (mostly!) ***analog***. For instance, air pressure can take on a continuum of values. So can acceleration and temperature. By contrast, computers speak only in ***digital***. So we need some 'translators'. This what the role of analog-to-digital converters (***ADCs***) and digital-to-analog converters (***DACs***).

ADCs and DACs in the Real World

1. **A Good Resolution: Under Pressure** Imagine you are driving up to the Blue Ridge Parkway on a fine early spring day. Just for fun, you bring a pressure sensor along with you. The pressure sensor outputs a voltage from 0 - 3.3V corresponding to pressures of 950 millibars (hurricane weather) to 1050 millibars (high pressure system = sunny day!). As you drive from campus up to the parkway, the pressure decreases from 1030 to 1000 millibars linearly over the course of 30 min (pressure goes down with altitude, and a storm front might push through tomorrow). The pressure sensor is connected to a home-built 4-bit ADC (because you were curious to build one yourself after studying them in Electronics), which also operates as a 3.3 V device.

   (a) How many different digital codes are possible with your 4-bit ADC?

   (b) Compute the resolution of your ADC—i.e. what is the discrete step size between each successive digital code?

   (c) Plot the pressure vs. time per the problem statement.

   (d) Assume you take data points at t = 0, at 3 min intervals up until 30 min. Overlay data points showing the values 4-bit ADC will record for the pressure signal vs. time.

   (e) Compare and contrast the actual pressure vs. the recorded pressure. Would you consider the ADC to be accurate or not?

2. **A Good Resolution: Bottle Band**

   Imagine the Bottle Boys are playing a hit concert and you are trying to record their music. Let's say they are simultaneously playing two pitches (perhaps a minor third in *Despacito*).

   This music generates a waveform that is a superposition of 2 sine waves + a DC offset, illustrated in Figure 2:

   $$a(t) = 1.5 + 0.5\cos(2\pi 200t) + 1.0\sin(2\pi 50t) \tag{1}$$

   Assume you record audio at a sampling rate of $f_s = 4000$ Hz (note this is 20× higher than the fastest 200 Hz component audio wave, way above the Nyquist rate).

   (a) Let's say you have a 1-bit ADC which is a 3.3V device. Plot the signal the ADC will record overlaid with the actual audio signal in Figure 2.

Figure 1: The Bottle Boys play music with recycled bottles! See them in action at this link. Image credit ntd.tv

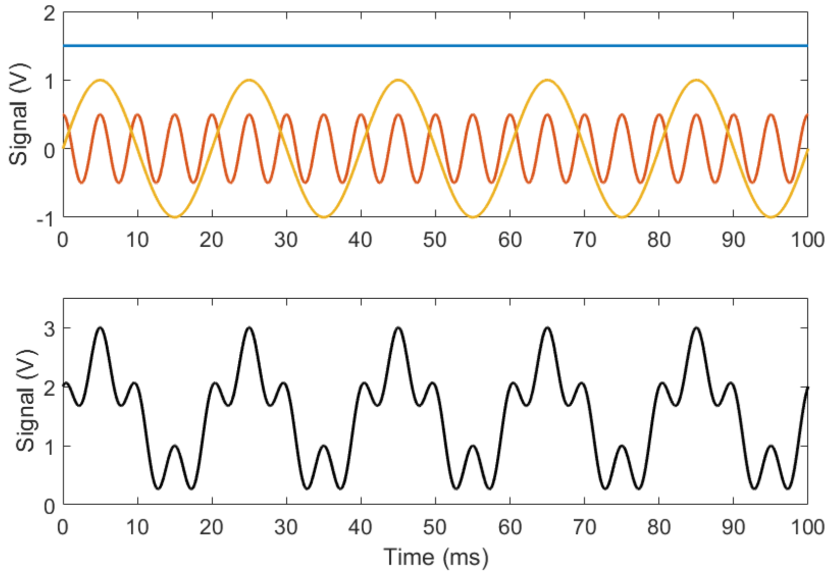

Figure 2: Top: composite functions of DC offset and 2 sine waves; Bottom: audio signal is the superposition of composite functions.

(b) Repeat part a, but assuming you now have a 2-bit ADC. Estimate the average error in the actual vs. recorded sound signal.

(c) How bits are required such that error between the recorded and actual signal is required to be ≤ 1 part in 1000 (equivalently, ≤ 3.3 V/1000 = 3.3 mV).

(d) Lastly, confirm your results by using a very slick interactive tool 13.1 on Dr. Valvano's e-textbook website from UT-Austin EE dept (scroll down about 1/5 of the way).

(e) Lastly, let's say you have a fancy mp3 audio player with a 16-bit DAC to generate audio samples at a rate of 44.1 kSamples/s. What is the resolution of this device? Can it accurately reproduce the audio waveform you recorded with the 10-bit ADC? Justify your answer.

3. **Look into your heart: Nyquist rate**

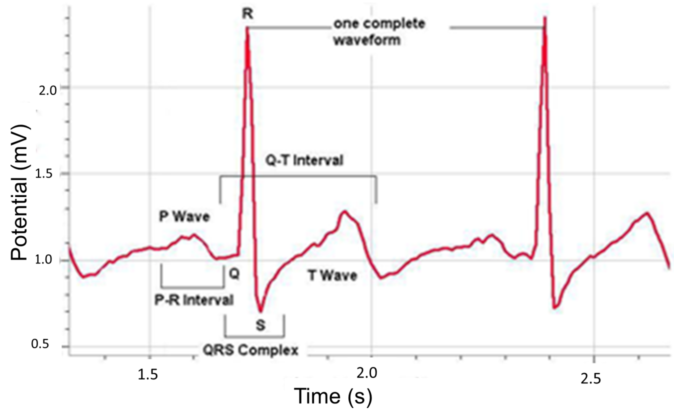

Figure 3: Electrocardiogram (ECG) signal. The P wave is generated contraction of the atria. The prominent QRS complex corresponds to contraction of the large, muscular ventricles. The T wave is formed when the ventricles repolarize (getting ready for the next heart beat).

Figure 3 shows an idealized electrocardiogram (ECG) signal, an electrical measure of heart activity.

(a) Measure the beat-to-beat intervals T_{beat} and corresponding frequency f_{beat}. Measure/approximate the timescale of the QRS complex T_{QRS}, and compute the equivalent (Fourier frequency) $f_{QRS} = 1/T_{QRS}$.

(b) Assume you have a 16-bit DAC with a sampling rate of $f_s = 2$ Hz. Carefully mark by hand the data points that will be taken, starting with the first sample at 1.5 s. Then connect them together to see what the recorded waveform will look like. Does the recording accurately represent the underlying ECG waveform? What features does it appropriately capture? Which ones does it miss?

(c) Repeat part a, but this time using a sampling rate of $f_s = 4$ Hz.

(d) What would select as the sampling rate in order to accurately capture the QRS complex. Use your answer from part a to fully justify.

4. **Welcome to the New School** Lastly, let's consider the resolution of a real device, namely the Intan RHD2000 chip which has taken the electrophysiology world by storm. The device has an 32 separate amplifiers + filters (***multiplexed***) to a 16-bit ADC. All of this electronics is integrated into a single chip smaller than a penny (amazing!).

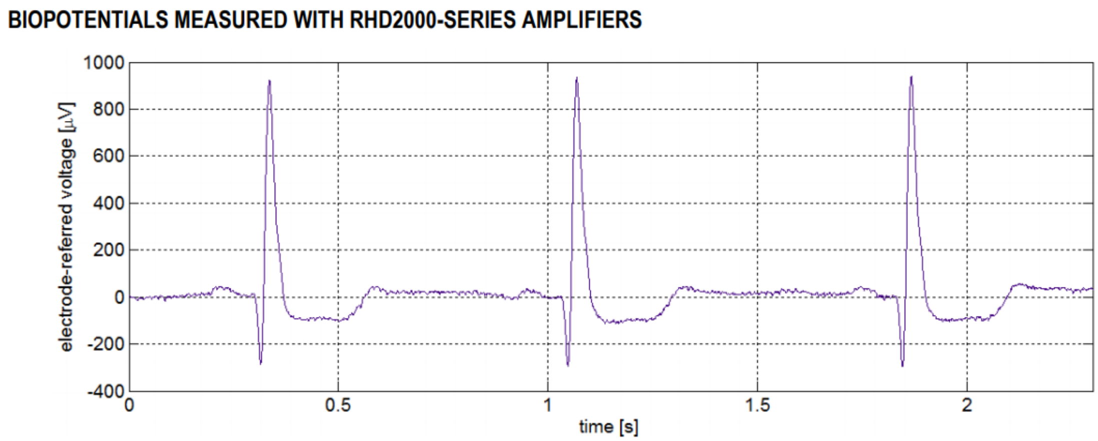

Figure 4. EKG signal recorded with RHD2216 using three Ag/AgCl electrodes (**in0+** and **in0–** on chest, 5 cm apart; ground on elbow). Amplifier was configured with f_L = 0.1 Hz, f_H = 100 Hz, and DSP high-pass filter set to 0.6 Hz. ADC sampling rate was 2 kS/s per channel.

Figure 4: Intan Technologies example recording of the ECG signal. Image credit, page 9 of Intan RHD2000 datasheet

(a) The Intan chip ADC essentially has a working range of ±5 mV. What is the smallest voltage step the ADC can resolve, in units of μV?

(b) In light of the ECG recording presented in Figure 4 compared to the idealized waveform in Figure 3, is a 16-bit ADC sufficient? Justify why or why not?

(c) Other companies make ECG hardware with 24 bit ADCs. What is the smallest voltage step that can be resolved (i.e. the resolution)? Are there any practical gains to be had with a 24-bit ADC, or do you think it is overkill? Justify why or why not. Of course the higher the number of bits, the higher the cost and power consumption.

(d) Finally, let's think about speed. The Intan RHD chips have a SPI interface with operating with a 24 MHz clock. One bit is transferred on each clock positive edge. 16 bits have to be transferred for a single sample of the ECG waveform on a single channel. What is the maximum possible sampling rate? Express your answer in MSamples/sec (M for mega).