Assignment # 3: Crossing That Bridge
The Wheatstone Bridge—Thermal Warning System
ENGN/PHYS 207—Fall 2019

Circuits You'll Build

- Standard Wheatstone Bridge
- Thermal Warning System

Skills and Concepts You'll Learn

- Wheatstone Bridge principle and practical operation
- All about thermistors—aka temperature sensitive resistors
- More Arduino programming and interfacing

1 The Wheatstone Bridge

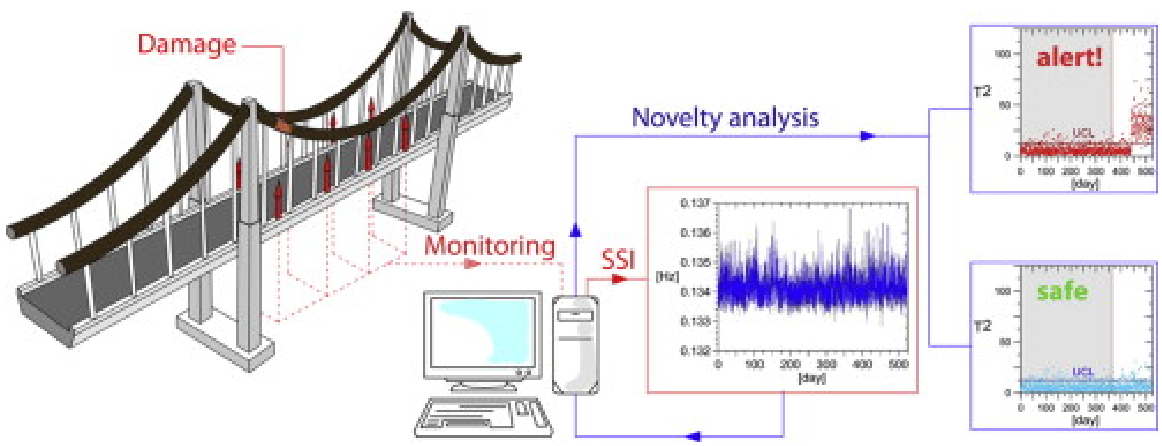

Figure 1: Structural Health Monitoring Example. Sensors on a suspension bridge measure vibrations in the bridge. Further analyses determine if the bridge is safe or not. Image credit: Comanducci et al. Journal of Wind Engineering and Industrial Aerodynamics, 2015: 141, 12-26.

1.1 Introduction

The **Wheatstone Bridge**[1] shown in Figure 2 is a clever 4 resistor configuration that **precisely measures small changes in resistance**[2]. While it was invented circa 1833 (people have been

[1] Interesting historical side note: Wheatstone didn't actually invent this circuit. Credit for the first description of the circuit goes to S.H. Christie; but Wheatstone is the one who found widespread practical use for this circuit.

[2] At this point, you might wonder "Why use a WB instead of a simple and beloved voltage divider? Great question! A full discussion can be found on this excellent post at EE world; scroll down to "I still don't see the cancellation effect, can you explain?"

clever for a very long time...), it still finds widespread use today in many engineering applications—mechanical, aerospace, and civil engineering, to name a few. Today, we'll build a thermal warning system. In the near future, we'll build vibration sensors with the YMCA after school program!

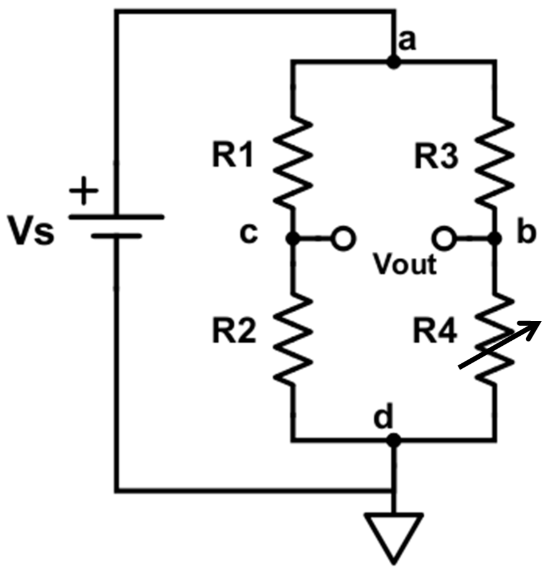

Figure 2: Basic Wheatstone Bridge Circuit. The voltage source V_s provides excitation at nodes a and d. R_4 is a variable resistor. The output is measured as the voltage difference at node b relative to node c.

The WB consists of four resistors in a series-parallel configuration, "excited" by a constant voltage source (e.g., power supply or battery). The output of the circuit is measured as the voltage difference across nodes b and c:

$$V_{out} = V_b - V_c = (V_b - V_d) - (V_c - V_d).$$

Note that node d is marked as ***ground***, the point in the circuit designated as 0 V.
The bridge is said to be ***balanced*** when $V_{out} = 0$. Starting in balance, changing the value of R_4 will make $V_{out} \neq 0$. In practice, R_4 might be a strain gauge, thermistor, flexible resistor—anything that changes resistance in response to some physical or environmental variable. As we'll see shortly V_{out} varies *linearly* with small changes in R_4.

1.2 Theoretical Considerations

1. The bridge output is defined as the voltage difference between nodes b and c: $V_{out} = V_b - V_c$. On which respective nodes would you place the (+) and (-) probes of the volt meter?

2. Show that:

$$V_{out} = \frac{R_1 R_4 - R_2 R_3}{(R_1 + R_2)(R_3 + R_4)} V_s \tag{1}$$

   Hint: Think voltage dividers. There are two loops total that include the source V_s.

3. The bridge is said to be ***balanced*** when $V_{out} = 0$. What is the mathematical relation for the resistances R_1, R_2, R_3, R_4 when the bridge is balanced?

4. Assume all resistors in your WB are initially equal $R_1 = R_2 = R_3 = R_4 = R$, so your bridge is balanced ($V_{out} = 0$). Then let R_4 increase its resistance by a relatively small amount: $R_4 \to R + \Delta R$. (The other resistors are constant in value: $R_1 = R_2 = R_3 = R$.) For the case that $\frac{\Delta R}{R} << 1$, show that:

$$V_{out} \approx \left(\frac{V_s}{4}\right)\left(\frac{\Delta R}{R}\right) \tag{2}$$

The term $\frac{\Delta R}{R}$ represents the relative change in resistance. For instance, if R_4 started out in the balanced configuration as 1000 Ω, and then changed to 1100 Ω, this is a 10% change in resistance, equivalently $\Delta R/R = 100/1000 = 0.1$. Note: if R_4 changes to 900 Ω, we have a -10% change in resistance, so we'll get a negative value for V_{out}.

5. Make a quick sketch of V_{out} vs. ΔR. Assume that $V_s = 5$V and $R = 1$ kΩ. What is the slope of this line? What would the slope of the line be if we changed R to 100 kΩ. Be sure to include positive and negative ΔR values in your sketches.

6. What is the maximum value of $\left|\frac{\Delta R}{R}\right|$ for which the linear approximation made in deriving Eqn 2 should be reasonably accurate? Briefly justify.

7. As circuit designers, we could choose R to be massive such that the assumption of$\frac{\Delta R}{R} \leq 1$ is easily realized in practice. The benefit of doing so: this makes for a nice linear relationship. But what's the drawback? For example, what's the output voltage we measure for V_{out} when we have a strain gauge that changes its value by 0.02% when the bridge shakes. Compare and contrast with the having smaller value for R such fractional change in resistance in a strain gauge is now 2%. Which is easier to measure?

2 Experiments/Circuit Design

2.1 The WB: Build a Bridge

"Is the section title really involve The WB?," you ask. Good news: No Dawson's Creek, Buffy the Vampire Slayer, or other low-quality re-runs here. Just the good ole Wheatstone Bridge.

1. Build the bridge (see Fig. 2). Set $V_s = +5$ V. Use 1 kΩ resistors for R_1, R_2, and R_3. (Remember to carefully measure and record the actual resistance of each of these.) For R_4, use a $\approx$2 kΩ pot. Balance your bridge by adjusting the value of R_4. Carefully measure and record the resistance of the pot with your circuit balanced. This is the reference value against which you compute the change in resistance. For example, if R_4 is 990Ω when the bridge is balanced, then you turn the pot value to be 940 ohms, this is means $\Delta R = -50\ \Omega$.

2. Imagine R_4 to be an environmental monitor, perhaps a soil moisture gauge whose resistance changes with water content. Or perhaps R_4 represents a strain gauge element attached to an airplane wing to measure its vibrations during flight. If the wing flexes up- or downward, the resistance will change by an amount ΔR. Or maybe its a light sensitive resistor to measure subtle fluctuations in light intensity for an optics experiment.

   We'll simulate this for now by turning the dial on the 2 kΩ pot. Sweep the pot through a range of resistance from 500 – 1500 Ω, in increments of $\approx$ 100 Ω. Carefully measure and record R_4 (pot value) and V_{out} for each. Make a plot of V_{out} vs. ΔR. Remember: ΔR is the ***change*** in the resistance of R_4 relative to the balanced condition. (See above example in part a)

3. Plot, analyze, and discuss your results in the context of Eqn 2. Is the relationship V_{out} and ΔR linear? Over the entire range of ΔR? Or are there seemingly non-linear regions? Annotate your plot to demarcate the linear and non-linear regions. There is no hard and fast cutoff; this is your judgment call. In your analysis/discussion, carefully consider the validity of the assumptions made when deriving Eqn 2.

2.2 It's Getting Hot In Here: Thermal Warning System

That's right, cue the music, Nelly circa 2002! Now that you've seen a bridge in action, let's build a thermal warning system. The core idea is to use a thermistor as one resistor in the WB. A thermistor is an circuit element with a temperature-dependent resistance. If only Isaac Newton had one back in the day! (See Figure 3).

Figure 3: Newton's Lab Fire (top) and thermistor element (bottom). The legend goes that Newton's dog knocked over a candle, setting fire to his experiment notebook. Another account says that he left a candle burning whilst he stepped out for a moment. A gust of wind blew it over, starting a fire in his lab. If only he had a thermal warning system! Image source: `https://www.alamy.com/stock-photo-isaac-newtons-lab-fire-18th-century-135096253.html`

The **design objective** at hand: Build a thermal warning system based on a thermistor incorporated into a Wheatstone Bridge configuration. An Arduino can be used to measure to WB output voltage, which can be used to solve for the resistance of the thermistor. And thermistor resistance can be used to solve for the temperature of the thermistor. When the thermistor temperature increases above a certain threshold (e.g. 40 C = 313 K), the Arduino will sound the alarm! The alarm is to consist of a buzzer sound and a blinking warning light (LED). On additional feature your system should include is a real-time graph of the temperature vs. time.

We'll be using a 10k thermistor, meaning the nominal resistance at room temp (25C) is $R_{25} = 10$ kΩ. Of course, you should measure the actual value at room temp. This particular thermistor is a "Negative Temperature Coefficient" (NTC) type, meaning the resistance decreases with increasing temperature.

To first approximation the resistance of the thermistor R_T is given by (in units of Ω):

$$R_T = R_{25} \exp\left[B\left(\frac{1}{T} - \frac{1}{298.15}\right)\right] \tag{3}$$

With a little bit of algebra, we can rearrange Eqn 3 to solve for the temperature (units of K) as a function of resistance:

$$T = \left[\frac{1}{298.15} + \frac{1}{B} \ln \frac{R_T}{R_{25}}\right]^{-1} \tag{4}$$

In Eqns 3 and 4, the B value (in units of K) is a material constant that can be found in the manufacturers data sheet (see course website). A further explanation of thermistor resistance vs. temperature can be found on page 4 in a technical note supplied by the manufacturer, Vishay.

As for the ***sound buzzer***, use a ***piezoelectric*** speaker. Piezoelectrics are special materials that convert a voltage into a physical deformation or vice versa. If we apply a square wave of voltage across the piezo material, it will respond in kind by vibrating at the frequency of the square wave (e.g. 500 Hz). Virtually all small speakers are made from piezoelectric materials. It is always a good idea to include a current limiting resistor ($\approx 100\ \Omega$) in series with the speaker to prevent it from burning out. You can see an example on this Buzzer tutorialArduino Project Hub Post. Note the usage of Arduino's tone() and noTone() functions. A single line of code can turn on or off the speaker—just amazing!

For ***real-time plotting*** of the temperature vs. time, consult this wonderful tutorial from adafruit demonstrating the Arduino Serial Plotter feature

For ***testing and validation*** purposes, you can use your fingers for a gentle heat source. For more fire power (pun intended ha!), use a heat gun. Be careful and mindful while using it not to melt or destroy other components.

3 More Arduino: Some tips on using it in the WB thermal sensing system

Below you'll find some tips on programming and using the Arduino, specific to the experiment d'jour.

1. Use the 5V and GND power ports on the Arduino to power the WB.

2. Remember the WB output is measured as the difference between the voltages at nodes b and c. Each Arduino analog input measures a single node in the circuit relative to ground. All measurements are made with respect to Arduino's ground (GND). So you'll need TWO separate analog reads to measure the bridge output V_{out}. For instance, connect node b on the WB to Arduino pin A0; and connect node c on the WB to Arduino pin A1.

3. Remember the analogRead() command reads in a 10-bit number (0-1023). Stupid computers, they only know 0's and 1's. We want an actual voltage. Furthermore, we know a reading of 1023 corresponds to 5V and a reading of 0 corresponds to 0 V. So there is a conversion factor of 5.0/1023 going from a 10-bit analog read to actual voltage. So you might do something like this in your code:

   ```
   int nodeB = analogRead(A0);
   int nodeC = analogRead(A1);
   float Vb = nodeB*(5.0/1023);
   float Vc = nodeC*(5.0/1023);
   float Vout = Vb - Vc;
   ```

   See how easy it is to program the Arduino? :)

   Note: lines of code such as the `float Vb` type tells Arduino "Please declare a 32-bit block of memory to store a variable we are calling `Vb`." An `int` declares a 16-bit block of memory and assign it a value of what's computed on the RHS of the equation.

4. You'll still need to write a few more lines of code to compute the change in resistance ($\Delta R/R$), based on the WB output voltage (V_{out} being measured by the Arduino, and also compute the temperature based on Eqn 4. Note: the log() function in Arduino and C++ computes the natural log. The log() function is not native to the Arduino core function library, so you'll need to include the math library. Toward the very tippy top of your code, you'll need to do this: `#include <math.h>`

   This tells the compiler, please also use the math library so we can use functions like log(). Note the use of the hash tag (the ORIGINAL use, take that Twitter bird!). Also, note there is NO semicolon termination.

4 What to Turn In

4.1 Basic Wheatstone Bridge Experiment with Potentiometer

Create a pretty graphic to summarize findings V_{out} vs. ΔR. Overlay empirical data with a line showing the theoretic/expected linearized behavior (see Eqn 2). You could additionally plot the linear best-fit to the data points for comparison. Written text should address when and how well theory and experiment agreed (or not). Pay particular attention to if and when the WB output is well approximated as linear. This section must not exceed 2 pages total! Probably a single page should suffice. Keep it clean, clear, correct, concise.

4.2 Thermal Warning System

Include also a brief Introduction highlighting the purpose of the circuit and intended applications. In the Methods sections, you should justify, in intuitive and quantitative terms, the design concept and rationale for all circuit elements integrated into your design. The Results should describe with words and beautiful graphic(s)/illustration(s) the proof-of-concept validation for your thermal warning system. In the Discussion, talk about the benefits and and limitations of your current device. How would you improve a future version of the system. 3 pages max! Include Arduino code in a separate Appendix, single spaced (does NOT count toward page limit).