Assignment # 2
The Wheatstone Bridge—Practical Applications
ENGN/PHYS 207—Fall 2018

What You'll Build and Test

- Wheatstone Bridge (the ubiquitous circuit sensing element)
- Pencil graphite resistor
- Real-world sensing system: Arduino with mechanical strain sensor

Skills and Concepts You'll Learn

- Wheatstone Bridge operational principle
- Pencil-drawn strain gauge form and function
- Basic Arduino programming and interfacing

1 The Wheatstone Bridge

1.1 Introduction

The Wheatstone Bridge[1] shown in Figure 1 is basically a sensitive resistance measurement device. Thought it was invented circa 1833 (people have been clever for a very long time...), it still finds widespread use today in many engineering applications—mechanical, aerospace, and civil engineering, to name a few. The basic idea is that the one of the bridge legs has a variable resistance (R_4, in this case). For example, the variable leg of the bridge could be a vibration sensor (aka strain gauge).

The WB consists of four resistors in a series-parallel configuration, "excited" by a constant voltage source (e.g., power supply). The output of the circuit is measured as the voltage difference across nodes b and c:

$$V_{out} = V_b - V_c = (V_b - V_d) - (V_c - V_d).$$

Note that node d is marked as ***ground***, the point in the circuit designated as 0 V.

The bridge is said to be ***balanced*** when $V_{out} = 0$. Starting in balance, changing the value of R_4 will make $V_{out} \neq 0$. In practice, R_4 might be a strain gauge, thermocouple, flexible resistor—anything that transduces one physical property into a change in resistance, hence into a change in output voltage!

1.2 Theoretical Considerations

1. The bridge output is defined as the voltage difference between nodes b and c: $V_{out} = V_b - V_c$. On which respective nodes would you place the (+) and (-) probes of the volt meter?

[1] Interesting historical side note: Wheatstone didn't actually invent this circuit. Credit for the first description of the circuit goes to S.H. Christie; but Wheatstone is the one who found widespread practical use for this circuit.

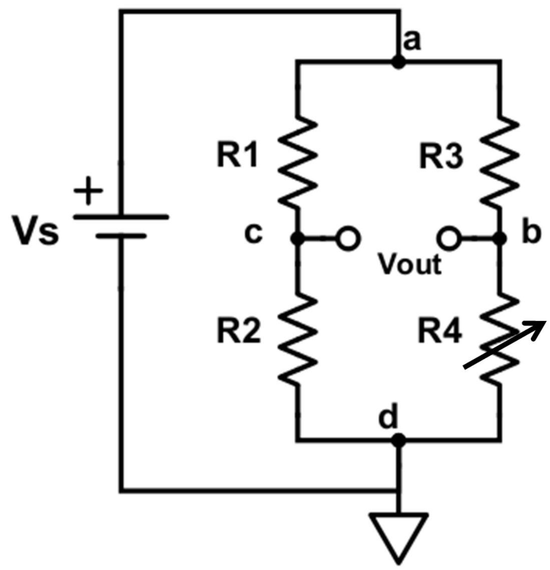

Figure 1: Basic Wheatstone Bridge Circuit. The voltage source V_s provides excitation at nodes a and d. R_4 is a variable resistor. The output is measured as the voltage difference at node b relative to node c.

2. Show that:

$$V_{out} = \frac{R_1 R_4 - R_2 R_3}{(R_1 + R_2)(R_3 + R_4)} V_s \tag{1}$$

   Hint: Think voltage dividers, one loop around each "branch" of the circuit.

3. The bridge is said to be ***balanced*** when $V_{out} = 0$. What is the relationship between the resistances R_1, R_2, R_3, R_4 when the bridge is balanced?

4. Assume all resistors in your WB are initially equal $R_1 = R_2 = R_3 = R_4 = R$, so your bridge is balanced ($V_{out} = 0$). Then let R_4 increase its resistance by a relatively small amount: $R_4 \rightarrow R + \Delta R$. For the case that $\frac{\Delta R}{R} << 1$, show that:

$$V_{out} \approx \frac{\Delta R}{4R} V_s \tag{2}$$

5. Make a quick sketch of V_{out} vs. ΔR. What is the slope of this line?

6. What is the maximum value of $\left|\frac{\Delta R}{R}\right|$ for which the linear approximation made in Eqn 2 remains reasonably accurate? Briefly justify. What are the practical considerations and/or consequences of trying to limit $\frac{\Delta R}{R} \leq 1$? In other words, what are the design considerations? What happens if this ratio exceeds 1?

1.3 Building Bridges: Getting Acquainted with the WB

"Is the section title really involve The WB?," you ask. Good news: No Dawson's Creek, Buffy the Vampire Slayer, or other low-quality re-runs here. Just the good ole Wheatstone Bridge.

1. Build the bridge (see Fig. 1). Set $V_s = +5$ V. Use 1 kΩ resistors for R_1, R_2, and R_3. (Remember to carefully measure and record the actual resistance of each of these.) For R_4, use a $\approx$2 kΩ pot. Balance your bridge. Carefully measure and record the resistance of the pot with your circuit balanced. How do your measurements compare to the expected result developed in question 3 above?

2. Imagine R_4 to be an environmental monitor, perhaps a soil moisture gauge whose resistance changes with water content. Or perhaps R_4 represents a strain gauge element attached to an airplane wing to measure its vibrations during flight. If the wing flexes up- or downward, the resistance will change by an amount ΔR. Or maybe its a light sensitive resistor to measure subtle fluctuations in light intensity for an optics experiment.

   We'll simulate this for now by turning the dial on the 2 kΩ pot. Sweep the pot through a range of resistance from 500 − 1500 Ω, in increments of $\approx$ 100 Ω. Carefully measure and record R_4 (pot value) and V_{out} for each. Make a plot of V_{out} vs. ΔR. Remember: ΔR is the ***change*** in the resistance of R_4 relative to the balanced condition.

3. Analyze and discuss your results in the context of Eqn 2. Is the graph linear? Over the entire range of ΔR? Or are there seemingly non-linear regions? In your analysis/discussion, carefully consider the validity of the assumptions made when deriving Eqn 2.

2 Practical Applications of a Pencil Graphite Resistor Sensing Element

Before you proceed with the lab, make sure you have perused the journal article by Liao et al[2] provided to you. It's a wonderfully detailed description of pencil resistors being used to measure mechanical deformations! You can learn some about circuits, and also about excellent scientific communication! (The article reads like a bedtime novel.)

2.1 Make and measure a pencil resistor

First well build pencil drawn variable resistors; the resistance value changes primarily as a result of mechanical deformation. Such resistors are inexpensive and easy to fabricate. As well, they lend themselves very well to all sorts of fun applications. But first, time to build a pencil resistor!
One needs nothing more than a pencil and paper. The key element is the graphite, which is electrically conductive. Its microstructure forms small non-conductive gaps when in tension, and generally better conducting platelets when in compression. The resistance of graphite varies depending on the grade of pencil lead, which is graded as illustrated in Figure 2.

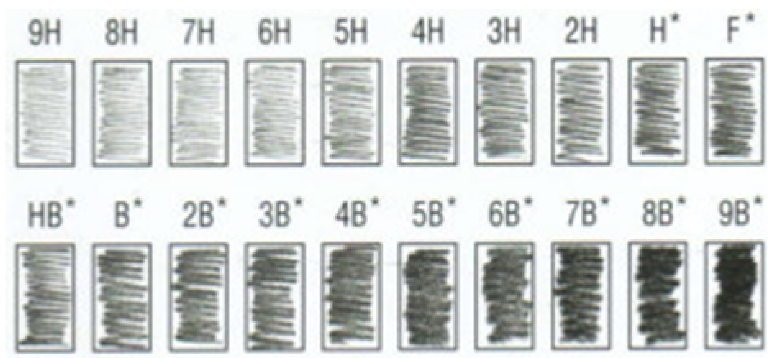

Figure 2: Different grades of pencil lead. Image credit: `http://www.efxkits.us/pencil-resistors-working-with-simple-project/`

For a first go, build a basic pencil resistor on paper. Draw a thick, straight line several inches in length on a piece of paper. Measure the resistance between two points of contact as a function of distance between the two points of contact. Do this systematically so that you can chart out length vs. resistance. Is the trend linear? Make a plot of resistance vs length to find out. This will also inform you essentially how a pot works on the inside.

2.2 Mechanical deformation—pencil resistor as pot

Similar to the first experiment, draw a resistor on paper, but this time, feel free to make some non-straight line arbitrary shape or shapes. Serpentine S-shapes often work well, but hey, its your resistor, so draw whatever you wish. Heart-shaped resistor? Sure, why not! Next, firmly attach two points of contact using either alligator clips or copper tape. The main idea is to make an electrical connection to your two defined points on your resistor so that resistance can easily be measured by an external device. One the connection is secured, bend and flex your pencil-on-paper resistor in

[2]Liao X et al. Flexible and Highly Sensitive Strain Sensors Fabricated by Pencil Drawn for Wearable Monitor. *Advanced Functional Materials*, 2015, 25, 2395 - 2401. Article can be accessed through W&L network connection/library at: `https://onlinelibrary.wiley.com/doi/pdf/10.1002/adfm.201500094`

some semi-methodical way. Make observations of how much the resistance varies for "small" and "large" deflections (qualitatively defined).
Next, incorporate your pencil resistor into the WB. Basically, the pencil resistor now acts as R_4 per Figure 1. You'll need to carefully select resistors R_1, R_2, and R_3 to balance the bridge. It might make sense to make one of these three a standard ***10-turn pot*** to aid in balancing the bridge.

2.3 Hello, Arduino

The Arduino is a very user-friendly and versatile microcontroller. It is the darling of the Maker universe. Time to get acquainted (in case you aren't already). It's easy to get started!

Figure 3: The venerable Arduino Uno R3. Image credit: amazon.com

1. Navigate to the Arduino "Getting Started" page: https://www.arduino.cc/en/Guide/HomePage.
2. Scroll down to Install the Arduino Desktop IDE. Then click the link for your operating system.
3. Alternatively, you can use the web-based editor, but the instructor mildly discourages doing so for now.
4. You'll need to install board-specific drivers. We use Arduino UNO R3 boards. You will need to select the appropriate board from the list in the next step. Note: The board is typically listed just as "Uno."
5. To test that the install went AOK, attempt to run the Blink Demo. Using the Arduino IDE (integrated development environment), compile and upload the example under File > Examples > Basics > Blink). To compile and upload all at once, click the check mark button at upper left. You should hopefully see and LED blinking on and off once per second. This is the Arduino signaling that it is alive and well. Hooray!

2.4 Real World Sensor System: WB + Pencil Resistor + Arduino

This experiment makes a good and proper measurements using the combination of WB, pencil resistor, and Arduino. This should be pretty sweet.

1. Previously, you used the BK Precision lab power supply to power your WB. Disconnect those wires now.

2. Use the 5V and GND power ports on the Arduino to power the WB.
3. Verify with a multimeter that your bridge is still balanced. If not, bring it back into balance.
4. Next, we'll use Arduino's analog read capability to make measurements! Essentially Arduino has on board not 1, but 6 (!) multimeters. These are labeled as pins A0 thru A5, respectively. Important notes on Arduino limitations:
   (a) You can only apply between 0-5 V (with respect to GND) at these pins, else the Arduino will likely fry!
   (b) All measurements are made with respect to Arduino's ground (GND).
5. Now, let's use the Arduino to make measurements of the nodal voltages V_b and V_c. We'll then use Arduino's built in math capability (it's a mini-computer, after all!) to compute the WB output $V_{out} = V_b - V_c$. Note that limitation 2 above means we can't just directly send in V_{out} to the Arduino.
   (a) Connect node b on the WB to Arduino pin A0.
   (b) Connect node c on the SB to Arduino pin A1.
   (c) Load the example sketch for making analog reads: File > Examples >Basics > AnalogReadSerial.
   (d) Copy and paste contents of this sketch into a new sketch.
   (e) Save the new sketch as "WBexample.ino"
   (f) Modify the new sketch to read from analog input A1. First look for a line of code
   `int sensorValue = analogRead(A0);`
   Then add a similar line right below it
   `int sensorValue1 = analogRead(A1);`
   (g) The Arduino's `analogRead()` function represents the voltage it reads as a 10-bit number. This means the Arduino reads an integer in the range of 0 to 1023 to represent voltages between 0 and 5 V. So you need to scale those 10-bit binary codes by a calibration factor of 5.0/1023.0 in order to get back to physical units. For example, you can write a line of code like this to do the conversion: `float Vb = sensorValue * (5.0 / 1023.0);`
   Note: the `float Vb` type tells Arduino "Please declare a 32-bit block of memory to store a variable we are calling `Vb`." An `int` declares a 16-bit block of memory.
   (h) Write a similar line of code for the voltage at node c.
   (i) Next we need to compute the difference between these two readings. To do so, create a new line of code below the analog reads: `float Vout = Vb - Vc;`
   See how easy it is to program the Arduino? :)
   (j) Now you'll need to write one or two more lines of code to compute the percent change in resistance ($\Delta R/R$), based on the WB output voltage (V_{out} being measured by the Arduino.
   (k) Compile and upload the code. Open the serial monitor or plotter to view the output. You can open the monitor which prints text by clicking the magnifying glass at upper right or Tools > SerialMonitor. The serial plotter makes a handy-dandy little plot. Open it by going to Tools > SerialPlotter.

(l) Then do a quick proof-of-concept test: Mechanically deform your pencil drawn resistor while monitoring the measurement for V_{out}. One easy way to do this is to tape it to a ruler, then flick the ruler to oscillate. Can you clearly see the changes in resistance? What is the difference in V_{out} that you can achieve? This is called the ***dynamic range***.

(m) Lastly, perform a good and proper device validation. This means you will do a carefully controlled and quantitative test in which you measure resistance change $\Delta R/R$ as a function of mechanical deformation, a la Liao et al 2015. Choose your own adventure—Your choice of what exactly you want to do. You will likely find inspiration in the Liao et al article. **Your idea must be approved by the instructor before you proceed.**

Written Report

Your **written report should consist of 2 separate documents** for Section 1 and 2 above:

1. Section 1: standard WB. Herein, you should quickly detail findings of V_{out} vs. ΔR. Create a pretty graphic to summarize findings. Written text should address when and how well theory and experiment agreed (or not). Pay particular attention to if and when the WB output is well approximated as linear. This section must not exceed 2 pages total! Probably a single page should suffice. Keep it clean, clear, correct, concise.

2. Section 2: This section should describe the validation experiment for your pencil-drawn resistor sensor experiment. Intro, Methods, Results, and Discussion should be described in sufficient detail such that your intended application is easily comprehensible. Ditto for what you measured in lab and how you measured it. You should include a single high-quality figure to illustrate findings (Figures in Liao et al would be very good to study as a paradigm). Discussion should include promises and limitations of your current device. 4 pages is the absolute max allowed, with no smaller than 11 point text font. Please be sure to include your Arduino code in an appendix.