Band-pass filter design/analysis problem
ENGN/PHYS 207—Fall 2018

Strike up the Band: Band-pass filter
In lab last Thursday, you designed a band pass filter with application to an EMG circuit (measuring electric potential generated by muscle constraction.) The pass band was set for $\approx$ 10 to 400 Hz. Let's delve deeper into the analysis of band-pass filter design—noting that all principles covered here are applicable to any type of RC filter.

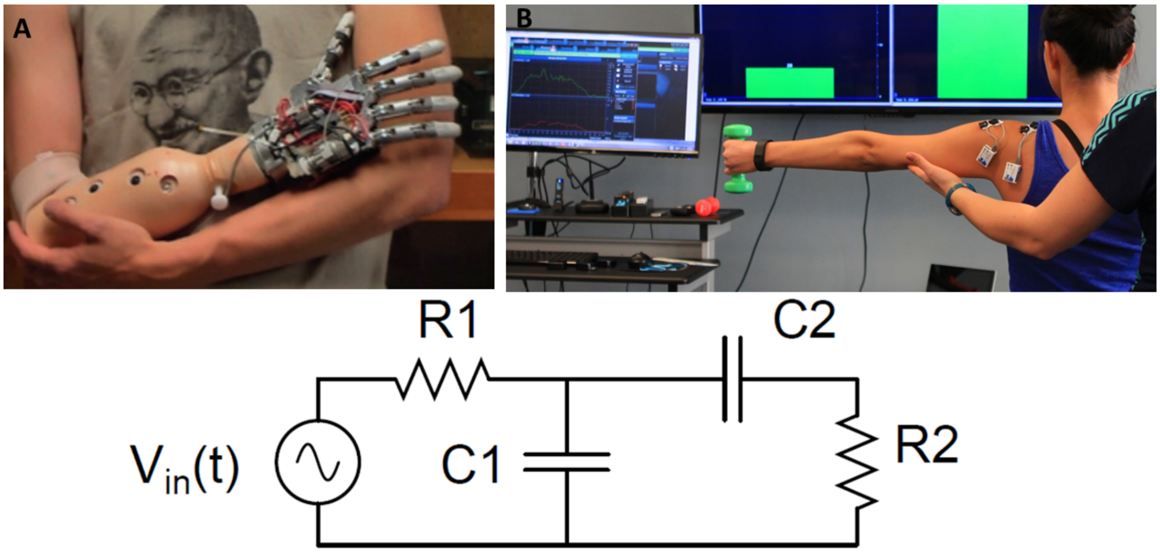

Figure 1: Band pass filter for surface EMG applications. Applications of sEMG include prosthetic devices and sports rehabilitation, to name a few. The simple band pass filter, consisting of cascaded 1-stage LPF and HPF is shown at bottom. The output is measured across R_2.

1. Diagram the BPF circuit in the limiting cases of very low and very high frequencies (e.g., $\omega \to 0$, $\omega \to \infty$). Argue why frequency components that are "too low" or "too high" are filtered out, as desired.

2. Now let's see why mid-range frequencies in the pass-band are indeed allowed to pass through to the output. Compute the magnitude of the imepdances $|Z_{C1}|$ and $|Z_{C2}|$ for $f = 100$ Hz (in the middle of your pass band). Accordingly, replace the caps in the circuit diagram with impedance elements of this magnitude. Note: units of $|Z|$ is Ω. Next, simplify your circuit using parallel and series equivalents to argue what the ratio of $|H(f)|$ will be at $f = 100$ Hz. Does this make intuitive sense?

3. Derive an *approximate* solution for the BPF's transfer function $\mathbf{H}(\omega) = \mathbf{V}_{out}/\mathbf{V}_{in}$.

4. Develop a sketch of the decibel gain $G(f)$ vs. $\log_{10} f$, spanning a frequency range of 0.1 - 40000 Hz (2 decades above and below the high and low filter cutoff frequencies.)

5. Using your approximate solution for the BPF transfer function, develop a sketch for $\phi(f)$ vs. $\log_{10} f$, spanning a frequency range of 0.1 - 40000 Hz.

6. In practice, sEMG recordings are subject to much higher levels of low frequency noise. Wiggles of the electrical cables during flexing, which typically occur at 1-5 Hz, are especially

problematic—we don't want those signal components in the sEMG signal. So we need to more strongly attenuate them (filter them out). **Make a new design for the band-pass filter that incorporates 3-stages total**: 2-stages to filter out the low-frequency noise sources, and 1-stage to filter out the unwanted high frequency noise sources. Carefully, consider the relation between the resistor values chosen for each cascaded section.

7. Write the approximate transfer function for this circuit. Develop a sketch for the decibel gain and phase shift spanning a frequency range of $\approx$ 0.1 - 40000 Hz.

8. Penultimately, let's take a step back from the "quick and dirty" solution—which almost always works in practice. Start back at the original 2-stage BPF (1-stage LPF plus 1-stage HPF). Write a sufficient set of equations from which the exact transfer function can be computed. The starting point is always to write KCL at nodes where current splits.

   It can be shown (with a little bit of algebra) that the exact solution can be written:

$$\mathbf{H}(\omega) = \frac{j\omega R_2 C_2}{1 + j\omega(R_1 C_1 + R_2 C_2 + R_1 C_2) - \omega^2 R_1 C_1 R_2 C_2}.$$

9. To convince yourself one last time that the 10× rule actually works really well in practice, compare the approximate and exact solutions. What is the extra term present in the exact solution not present in the approximate solution. What assumptions must you make about component values in order for the quick and dirty solution to be essentially equal to the exact solution?

10. Lastly, let's return to the time-domain—i.e. what effect the filter has on the actual waveforms vs. time—what you'd see on an oscilloscope for input vs. output. Consider a raw recorded signal $V_{raw}(t)$ to be a waveform that consists of sum of three cosines:

$$V_{raw}(t) = a_1 \cos(2\pi f_1 t + \phi_1) + a_2 \cos(2\pi f_2 t + \phi_1) + a_3 \cos(2\pi f_3 t + \phi_1)$$

    where $f_1 = 2$ Hz, $f_2 = 200$ Hz, $f_3 = 2000$ Hz; $a_1 = 3$ mV, $a_2 = 1$ mV $a_3 = 0.2$ mV and $\phi_1 = 0$, $\phi_2 = \pi/2$ rad; $\phi_3 = \pi$ rad .

    Compute the filtered signal $V_{filtered}(t)$. Plot the raw (unfiltered) and filtered signals vs. time. How well did your filter do in isolating just the muscle contraction components in the raw recorded signal? (Ultimately this is a scientific aesthetic question: How clean does your data really have to be?)